

Spring 5-13-2022

## Sheltered Math Curriculum for Middle School English Learners

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Sheltered Math Curriculum for  
Middle School English Learners

A Plan B Project Presented to  
the Graduate Faculty of  
Minnesota State University Moorhead

By

Jasmine Carolyn Ercink

In Partial Fulfillment of the  
Requirements for the Degree of  
Master of Arts in  
Teaching English as a Second Language

May 2022

Moorhead, Minnesota



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### **Abstract**

Language barriers have shown a need for differentiation and sheltered instruction in the classroom for English Learners (ELs) to be successful in the United States public school system. This project proposes a mathematics curriculum using SIOP so that both groups of students in the middle school level can increase their proficiency in the mathematics content area as well as experience opportunities for academic and social language development. The purpose of this report is to describe the processes, methods, data, and intent of the mathematics curriculum for these learners. The curriculum acts as an effective intervention to fill gaps in both the academic language of mathematics and the skills necessary to be successful with their grade level peers in a matter of one to two years.

*Keywords:* academic language, content, culture, English learners (ELs), proficiency

## **Chapter 1: Introduction**

As a math education major in college, I was privileged enough to discover a part time job opportunity that enabled me to mentor middle school and high school students who were struggling academically, socially, and behaviorally in school. The first student I was assigned in this program was a middle school student from Mexico. She struggled tremendously in her math class, and it was my job to help her understand her homework, try to give her additional materials to aid her understanding, as well as help her with her school homework. I quickly learned that her gaps did not come from a lack of mathematical knowledge but from a lack of the knowledge of the academic language in English behind mathematics. We worked extensively on understanding what each question was asking rather than how to solve it. This student made tremendous gains throughout the year that I assisted her in the after-school mentoring program, and it was then that I realized this was the type of student I most enjoyed working with.

Everyone's goal after graduating is to ultimately land a job in their field. After graduating college with my mathematics education degree, I, like everyone, interviewed for several middle school and high school mathematics positions. After a few interviews, I was intrigued to see a job teaching English Language Learner (ELL) mathematics in the middle school setting. I was also skeptical of this position, as I was not trained to be an ELL teacher, nor did I know what the job fully entailed. That week, I was offered two jobs – one as a middle school math teacher on an advanced Science-Technology-Engineering-Math (STEM) team, and one as an ELL math teacher, traveling between a high school and middle school. It was not hard for me to decide; I not only wanted to teach but also wanted to help. I took the ELL job.

### **The Context of the Study**

The district where I was hired is located in a medium-sized, urban, Midwestern city with a vibrant and growing metropolitan area. The community serves as a regional center of education, commerce, and health care and is said to hold an array of cultural influences and diversity as well as attractions and activities far beyond what is characteristic for a city its size. The school district offers a wide variety of athletics, clubs, and activities for all students. The district has also received recognition from several community organizations for its community service (Fargo, 2018).

In this district 21.7% of the students qualify for free or reduced lunches, 8.7% of the population is special needs and/or on an IEP, and 5.6% of the population is in the English Learner Program. The district now serves students who speak over 13 different languages other than English. Those with the most speakers are Nepali (23%), Somali (13%), Arabic (12%), and Spanish (11%). Most of the ELL students are also from lower class families and receive free or reduced lunches. Some ELL students may be on an Individualized Education Plan (IEP) through Special Education as well (Building Demographics, 2018).

According to the North Dakota Department of Public Instruction (NDDPI), all school districts in North Dakota are required to be prepared for English Learners who may enroll in their schools, and appropriate programming must be developed to meet the needs of the students to overcome the language barrier. The district where I work has chosen different program models for the varying grade levels and buildings but does meet the federal requirements for meeting these needs (NDDPI, 2021).

## Legislation

Over time, there have been federal level laws, policies, and major Supreme Court rulings affecting ELLs in the K-12 schools. These include the 14<sup>th</sup> Amendment to the Constitution of the United States in 1868, the May 25<sup>th</sup> Memorandum of 1970, the *Keyes v. Denver* Supreme Court case of 1973, the *Lau v. Nichols* Supreme Court case of 1974, the *Plyler v. Doe* Supreme Court case of 1982, the *Casteñeda v. Pickard* federal court case of 1978, the No Child Left Behind Act of 2001, the Dear Colleague Letter of May 2014, and the Every Student Succeeds Act (ESSA) of 2015.

As early as 1868, the 14<sup>th</sup> Amendment of the Constitution was passed in the United States. Simply stated, this amendment indicates that it does not matter if someone is a citizen of the United States or not, whether the person is here legally or illegally, all people are entitled to equal protection of the laws under the U.S. Constitution (U.S. Congress 14<sup>th</sup> Amendment). In 1982, an interpretation of the 14<sup>th</sup> Amendment was upheld in the *Plyler v. Doe* Supreme Court case. This interpretation clarified the language of the amendment ensuring that “equal protection of law” extends to all people within the territorial boundaries of a defined area. This applies directly to ELs in schools as it ensures that states cannot constitutionally deny students free public education on account of their immigration status. *Plyler v. Doe* safeguards equal access to education for children regardless of status.

The Civil Rights Act of 1964 (CDRC, 2021), which declares education is one of the areas of civil rights guaranteed for all and cannot be abridged due to race, color, or national origin, whether a person was born in the US or not or whether a person is a citizen of the US or not, entitles all to participate in activities and programs receiving funding from the Department of Education. This law aligns with the 14<sup>th</sup> Amendment and underlies the ruling in *Plyler vs. Doe*,

which affirms school officials may not require children to prove they are in this country legally by asking for documents such as green cards, citizenship papers, etc. They may only require proof that the child lives within the school district attendance zone, just as they might for any other child. The Civil Rights Act of 1964 require schools to take measures to ensure that English Learner students can fully participate in school programs and services as well.

The May 25 Memorandum of 1970 (CDRC, 2021) stated, “Where the inability to speak and understand the English language excludes national origin minority group children from effective participation in the educational program offered by a school district, the district must take affirmative steps to rectify the language deficiency in order to open its instructional program to these students.” And further, the Supreme Court agreed in its 1974 decision in *Lau v. Nichols* (Cornell Law School) that, “There is no equality of treatment merely by providing students with the same facilities, textbooks, teachers, and curriculum; for students who do not understand English are effectively foreclosed from any meaningful education” (NCELA, 2006). If an EL student does not understand the English language and in turn excludes effective participation in any educational program offered by the school district, said district must take affirmative steps to rectify the language deficiency in order to open its instructional program to these students. Additionally, ELs must not be assigned to any ability grouping or tracking system employed by the school system to deal with their language skill needs. School districts also have the responsibility to adequately notify parents of the ELs of school activities which are called to attention of other parents which may have to be provided in a language other than English in order to deem adequate.

In *Lau vs. Nichols*, the U.S. Supreme Court affirmed the Department of Education’s May 25, 1970, Memorandum, that directed school districts to take steps to help ELL students

overcome language barriers and to ensure that they can participate meaningfully in the districts' educational programs (CRDC, 2021).

In 2001, the No Child Left Behind (NCLB) Act became a law. Within NCLB, the regulations define a recently arrived Limited English Proficiency (LEP) student as one who has attended schools in the United States for less than 12 months, and as a student whom, during this time, a State may exempt from one administration of the State's reading/language arts assessment. For purposes of participation in a State's assessment system, recently arrived LEP students must take an English proficiency assessment. These details about the flexibility regarding assessment do not relieve schools from their responsibilities to serve LEP students or diminish the responsibility for schools to provide appropriate instruction to recently arrived LEP students so that they can gain English language skills and master content knowledge in reading and language arts, math, and science. This act reaffirms the responsibility of schools to teach ELLs both English language and other subject content.

Even as recently as 2014 in the Dear Colleague Letter of May 2014, changes were still being made to the current laws and policies to ensure ELL students are served properly. The Dear Colleague Letter clarifies that residency within a district is not the same as citizenship or legal status. Schools may require proof of residence, but not proof of citizenship or legal entrance. Those with proof of residence within the district must be served accordingly by law, which reconfirms Plyler v. Doe and the reading of the 14<sup>th</sup> Amendment as establishing the rights of persons, not of citizens. A district may require copies of phone or water bills or lease agreements to establish residency, but inquiring into students' citizenship or immigration status, or that of their parents or guardians would not be relevant to establishing residency within the district. In short, The Dear Colleague Letter of May 2014 reminds states, school districts, and

schools of their obligations under federal law to ensure that ELs have equal access to high-quality education and the opportunity to achieve their full academic potential. Districts must identify and assess EL students in need of language assistance in a timely, valid, and reliable manner; provide ELs with a language assistance program that is educationally sound and proven successful; sufficiently staff and support the language assistance programs for EL students; ensure ELs equal opportunities to meaningful participation in all curricular and extracurricular activities; avoid unnecessary segregation of EL students, ensure ELs with disabilities are evaluated in a timely and appropriate manner for special education and disability-related services and that their language needs are considered in evaluations and delivery of services; meet the needs of ELs who opt out of language assistance programs; monitor and evaluate ELs in language assistance programs to ensure their progress with respect to acquiring English proficiency and grade level core content; ensure meaningful communication with Limited English Proficiency parents (Sugarman, 2019).

In 2015, President Obama signed the Every Student Succeeds Act (ESSA), which includes important policies that recognize the needs and diversity of ELs in an effort to close the ongoing achievement gap between them and the other students. The purpose of this act was to replace and update the NCLB from 2002. The intent of the law has been to raise achievement for low-income and otherwise disadvantaged children. This also requires state's goals for its students' English language development must be linked to the level of English that students need to succeed in grade-level content instruction. This directs states to develop learning standards and corresponding annual assessments that measure student progress toward academic goals, including annual standardized assessments for English language arts and math for all students, but also English language proficiency for ELs. Finally, schools must use this data and evidence

to make decisions on whether instructional programs are effective and how to improve them if necessary (Sugarman, 2019).

In accordance with federal regulations as well as with the court cases and major rulings involving student rights, legislation in North Dakota requires all school districts to develop and report back to the state their plan to identify and serve ELs. This plan can also be referred to as a district's EL program plan. A district's EL program plan should include the following components: procedures for EL student identification, language assessment procedures, procedures for the placement of EL students, background on the LIEP/program models implemented in the district, identification of EL staff, EL program exit criteria, monitoring procedures, and program evaluation plans (NDDPI, 2021).

The state of North Dakota, along with thirty-nine other states, territories, and federal agencies in the country, belongs to the WIDA Consortium, which was established in 2002 to test and support English Learners. WIDA's mission "advances academic language development and academic achievement for children and youth who are culturally and linguistically diverse through high quality standards, assessments, research, and professional learning for educators" (WIDA, 2020). The consortium has developed and oversees a three-pronged system for schools working with ELs: a set of standards, a set of proficiency level descriptions, and a set of assessment instruments to measure proficiency.

WIDA initially wrote their ELP standards and model performance indicators to meet the requirements for NCLB, they also went beyond the requirements to help with the increasing need for academic language support for ELs. WIDA's goal in creating their ELP standards was to "push schools in the same direction, toward including ELLs in the grade-appropriate curriculum designed to maximize their learning of content while increasing their proficiency in English"

(Samson, 2012). The overarching goal of WIDA is to promote educational equity and academic achievement for linguistically and culturally diverse students. WIDA advances academic language development and academic achievement for children and youth who are linguistically diverse through high quality standards, assessments, research, and professional learning for educators. To achieve this goal, WIDA partners with districts, states, and national experts to conduct research focused on understanding and explaining the educational experiences and outcomes of language learners. This research informs the decision-making needs of educators and policymakers who serve ELs, in accordance with the federal government laws (WIDA Research Report, 2021).

The WIDA ACCESS test is the annual assessment for English Proficiency in the four language domains of listening, speaking, reading, and writing. The WIDA ACCESS test results are used to assess social and general academic English in the four language domains as well as language used in language arts, mathematics, science, and social studies. The test is broken down into grade level clusters (K, 1-2, 3-5, 6-8, and 9-12). Within each grade level cluster, the ELP standards are embedded, and each academic language area is represented.

WIDA has identified five English Language Development (ELD) Standard Statements to provide educators with a connection between language development and academic content area learning. The statements represent the broad and ever-present language of the disciplines (WIDA, 2020).

**English Language Development Standard 1:** English language learners communicate the **Social and Instructional** purposes within the school setting.

**English Language Development Standard 2:** English language learners communicate information, ideas, and concepts necessary for academic success in the content area of **Language Arts.**

**English Language Development Standard 3:** English language learners communicate information, ideas, and concepts necessary for academic success in the content area of **Mathematics.**

**English Language Development Standard 4:** English language learners communicate information, ideas, and concepts necessary for academic success in the content area of **Science.**

**English Language Development Standard 2:** English language learners communicate information, ideas, and concepts necessary for academic success in the content area of **Social Studies.** (pg. 9)

WIDA's ACCESS test results are interpreted to produce six levels of proficiency: Level 1 Entering, Level 2 Emerging, Level 3 Developing, Level 4 Expanding, Level 5 Bridging, and Level 6 Reaching (WIDA, 2020). The WIDA ACCESS Test gives the most accurate annual English Proficiency levels of each student in each language domain and their overall level. Domain and overall scores are used by teachers to place ELs into the correct ESL classes, to identify the appropriate level of accommodations in their mainstream classes, and in general to make decisions about students' education. Refer to the *ACCES for ELLs Interpretive Guide for Score Reports* for a full explanation of WIDA Proficiency Level Descriptors, Grades 1-12, in the domains of listening, reading, writing, and speaking (WIDA, 2022).

The state's membership in the WIDA consortium provides every district in the state, large and small, the necessary tools to meet federal requirements to identify, place, regularly

assess, and exit ELs in their program in a consistent way across the state and meet the goals in their ESL Program plans. Nevertheless, each district retains the flexibility to choose the type of English Language Program they will offer to their students and essentially what works best for their ratio of EL students to their EL teachers.

In the district where I teach, elementary students, grades K-5, operate on a pull-out schedule, where an EL teacher will take them from their mainstream classroom for approximately one hour and work on specific skills with the EL student to try to improve their English. This model allows schedule variations according to the case load of the teacher and the English proficiency of the EL student. In the district's high schools, four core subjects are designated for sheltered instruction specifically for EL students: science, social studies, math, and English (both writing and reading). High school students with a low English proficiency will be in an EL classroom for up to five or six hours a day, and they will be in mainstream classrooms for two other hours in the day for elective classes and other non-core classes, such as health and physical education. At the middle school where I teach, we currently only have designated teachers for sheltered instruction in the subjects of mathematics, English (which also includes both reading and writing classes), and a newly developed Introduction to Social Studies class.

## **SIOP**

The district I teach in has adopted the Sheltered Instruction Observation Protocol (SIOP) instructional approach, which aims to teach EL students grade-level content standards while simultaneously enhancing students' English proficiency. Content-based sheltered instruction is teaching grade-level subject matter in English solely for EL students. Sheltered instruction focuses on ways to make content comprehensible and attainable, and engage EL students

academically, while also consciously promoting English language development (need a source for this definition). Usually, these sheltered instruction classrooms are smaller than the average mainstream classroom and consist of only EL students. The goal is to work on English language skills and not fall behind in other academic areas. This is done by modifying the English instruction without compromising content using a variety of techniques such as the use of visual aids, modeling, demonstrations, graphic organizers, vocabulary reviews, predictions, adapted texts, cooperative learning, peer tutoring, etc. Sheltered instruction classrooms create a non-threatening environment where students feel comfortable taking risks (Reider, 2004). For now, the amount of sheltered instruction that the students receive gradually increases with their grade level as well as their English proficiency levels. The district has designed it so that the older the student is, the more sheltered instruction they will receive if their English proficiency is lower.

My EL math classes have students ranging from true beginner or new to English (i.e. no ability to speak, listen to, read, or write in English) to students who are nearing proficiencies close to their native English-speaking peers. Some students come to my classroom with little to no English skills, and some students also come to my classroom with little to no mathematics skills, but all served by the EL program. It is my job to group the students in classes according to their English and mathematic proficiency levels and teach them the necessary math skills and academic language in English to be successful in a mainstream class within one to three years. They may continue in an EL classroom setting in high school if their proficiencies remain low upon leaving the middle school. My curriculum must be accessible to all of the EL students from the bottom of this ladder to the top of it.

**Problem Statement**

The laws state that ELs must have equal access to high-quality education and the opportunity to achieve their full academic potential, and that the schools have a legal responsibility to teach ELs both English language and other subject content to reach that potential. In the district where I work, the mainstream mathematics curriculum addresses the needs of students who have attended school in the United States for the entirety of their primary education with English as their first language. The curriculum is largely culturally biased and leaves little to no room for intervention for those who have educational gaps in the mainstream curriculum pathway and standards. When I first started my EL Mathematics teaching job, there was already an EL program in place, and sheltered instruction in mathematics was available to EL students. However, I was given a curriculum that the other EL high school mathematics teachers used to teach their EL students. I tried to use this curriculum in the middle school, but I quickly realized that this material needed to be customized, adjusted, and modified to meet the needs of the middle school students I was teaching. Similarly, I definitely could not use the curriculum that the mainstream teachers were using at the middle school level, as it was far too language heavy and mathematically advanced for the EL students in my classes. I needed something new. That is when I came up with the idea for the project focused on finding effective curricular materials for teaching EL students mathematics in the middle school. The root of my project stems from English Learners' need for differentiation and sheltered instruction in the classroom to be successful in the United States public school system, and the need to enact differentiation in the middle level math classes.

In this project, I will discuss problems with current standards and content in the mainstream mathematics classrooms, the languages, and cultural differences that create learning

barriers for EL students, and solutions for these problems in a new mathematics curriculum. My project reflects only the sheltered instruction of EL mathematics at the middle school level.

Chapter 2 of this report covers the struggles EL students face with the mainstream mathematics curriculum. The third chapter covers the struggles EL students encounter with language and culture and the effect on mathematics. Chapter 4 describes and presents the process used to appropriately place EL students in the sheltered instruction classes and the curriculum I created to meet the needs of the students with these struggles in the EL mathematics sheltered instruction classroom. Chapter five explains and presents the curriculum that was created to meet the needs of the ELs in the middle school setting which will address both the language barriers and the educational gaps, vocabulary, and relevant content. The sixth and final chapter covers the steps moving forward with the curriculum that was created and plans to research its effectiveness for future revisions.

## **Chapter 2: Struggles with Mainstream Mathematics**

The state mandated curriculum provides educational standards, which are statements designed to describe a clear path for students to gain the proficiency required to learn increasingly complex material. Standards are what drive the K-12 curriculum, and the North Dakota Mathematics Content Standards maintain grade-level course continuity in mathematics throughout the state (NDDPI, 2019).

NDDPI states that the content standards include critical processes and proficiencies embedded within the content. The mathematical content standards provide rigor for student learning with conceptual understanding, procedural skill and fluency, and application. “The ability to apply mathematical knowledge correctly depends on students having solid conceptual understanding and procedural fluency.” (NDDPI, 2019).

As a result of these standards, math teachers have multiple responsibilities: help students develop the most important mathematical concepts; help them learn the processes to execute mathematical operations; help them identify mathematical concepts in everyday life and apply the mathematical operations to those everyday situations; and develop the academic language necessary to talk about these concepts and operations.

The district where I work follows the Common Core Standards for the mathematics curriculum, along with the grade level goals as published in *Everyday Mathematics* from The University of Chicago (Isaacs, 2018). The district’s curriculum also assumes that all students entering the sixth grade have learned and mastered the standards in kindergarten through 5<sup>th</sup>

grade prior to entering the middle school setting. For many students, especially English Learner students and New Americans, this is not the case.

The mathematics curriculum that the district uses in the elementary schools (K-5), (See Appendix A) is broken down by grade level goals, or scope. Within each elementary grade, K, 1, 2, 3, 4, and 5, shared topics are taught in mathematics: numbers and numeration, operations and computation, data and chance, measurement and reference frames, geometry, and patterns, Functions, and Algebra. With each subsequent year, the rigor of each topic increases. For example, in kindergarten the curriculum in the numbers and numeration will go as high as place value up to the hundreds and comparing numbers up to 20. When the students reach fifth grade, the curriculum in numbers and numeration includes all place values from the billions place down to the billionths place in decimals, as well as comparing and ordering whole numbers, mixed numbers, fractions, and decimals. Students in kindergarten will be learning to identify simple shapes and basic solid figures in geometry, and students in fifth grade will be learning properties of angles in special triangles and quadrilaterals and sketching examples of reflections, translations, and rotations.

But what if the student has not been in the American school system since kindergarten? What if a student did not have an education at all previous to entering the United States school system? Many students do not possess this previous knowledge, this foundation, to build on. Many students do not possess solid conceptual understanding or procedural fluency prior to arriving at a new school.

Immigrants, refugees, New Americans, and all EL students face a number of challenges while transitioning into their new school lives. Language, of course, is a major barrier for ELs, but it is not the only barrier. For EL children, learning English and assimilating to new culture

are two of the highest barriers to face while trying to accomplish a successful schooling career (J. Vecchiarelli, 2019). Some EL students even enter the United States school system without any previous education.

### **SLIFE**

Students with limited or interrupted formal education (SLIFE) generally come from a home in which a language other than English is spoken, have gaps in their education from their home country, and are below grade level in reading and mathematics. Some of these students may have attended school in the U.S. but still have gaps in language and literacy (Fenner, 2017). According to a research study conducted at John Hopkins University 2019, SLIFEs usually have experienced one of the following patterns: they are newcomers with two or more years of education interrupted in their native country; they may have attended school in the U.S. but returned to their native country for a period of time and then returned to the U.S. again; they may have attended Kindergarten in English, 1<sup>st</sup> and 2<sup>nd</sup> grade in their first language, and then back to the U.S. for 3<sup>rd</sup> grade; they may have attended school in several locations with several moves since arriving to the U.S. with possible weeks between schools and these several different schools trying to start over to meet their needs. Oftentimes these students may not have had an opportunity to attend school due to transience or a situation in their home country. SLIFEs may come from countries of poverty, disaster, or civil unrest that affect the development of literacy and opportunities for education. They may also come from countries where persecution or strict rules about gender, social class, or ethnicity prevented them from attending school. In short, students with limited or interrupted education, through no choice of their own, are starting school one or more years behind their grade-level peers (Robertson & Lafond, 2019). They are likely to need instruction in basic mathematical skills that their peers do not need.

SLIFEs may have other unique needs when it comes to formal education. They are likely to need additional support for a number of reasons such as post-traumatic stress disorder from events encountered during migration, war in their home country, or just the loss of stability from uprooting to move to the United States. Disadvantages due to socioeconomic circumstances may become apparent and a cause for additional support. Finally, frustration at being torn between excitement of finally attending school yet being behind their peers and needing to work harder to catch up brings another reason for extra support (Robertson, 2019).

I have designed a new curriculum for middle school EL students for multiple reasons: to help ELs work towards grade level proficiency, to help ELs overcome social, cultural, and linguistic boundaries, to help ELs develop foundational mathematical concepts and academic vocabulary, and to help ELs increase their English language proficiency concurrently with their general educational development.

### **Chapter 3: Struggles with Language and Culture**

When families arrive either as refugees or immigrants to the United States, there are many different labels given to the students depending on the state and school district to which they are attending, all of which hold the same relative meaning: that these students did not learn English as their first language. A few of the more common labels are English as a Second Language (ESL), English Language Learner (ELL), English Learner (EL), and Multilingual Learner (ML). Some of these students who did not learn English as their first language can be identified in the school setting and receive services from the ESL program if identified, although some do not. In order to qualify for services of an ESL program, a series of qualifications and identifications of limited English proficiency (LEP) must be met first. Students must speak a language other than English, they must be registered and enrolled in school, their limited English skills must prohibit them from participating fully in society and the classroom, and they must be between three and twenty-one years of age (Uro & Barrio, 2013). There are no federal regulations in place to determine whether the English skills of the EL student prohibit them from joining society and the classroom functionally, so each state is responsible for setting their own parameters.

#### **Identifying ELs**

Currently, 36 U.S. states, including North Dakota, follow the parameters set by the World-Class Instructional Design (WIDA) Consortium. However, WIDA Assessment is not the first or only parameter which students must meet. At this time in the district where I work, the following steps are taken to determine a student's eligibility to be classified as an EL student and

follow an Individualized Learning Plan (ILP) to meet their language needs. First, each new student completes a Home Language Survey (HLS) as a piece of their school registration paperwork; which asks the native language of the student, among other things. If a student indicates that their native language is not English, the EL certified staff will review the HLS, the student cumulative file, and interview the student and/or family. If the data and staff deem further testing is necessary, the student then takes an ELP screening assessment to determine their eligibility. The screening assessment is provided through WIDA. To identify EL students, Kindergarten students take the WIDA MODEL test, and Grades 1-12 students take the WIDA Screener test. If the student's test scores do not meet proficiency, they are admitted into the EL program and the EL staff will write an ILP. After being admitted into the EL program, the student will take a yearly proficiency test called the WIDA ACCESS for ELs Test which measures their proficiency in the four language domains: reading, writing, listening, and speaking, to determine growth and further admittance into the program. At the point in which a student has reached proficiency, they are exited from the program.

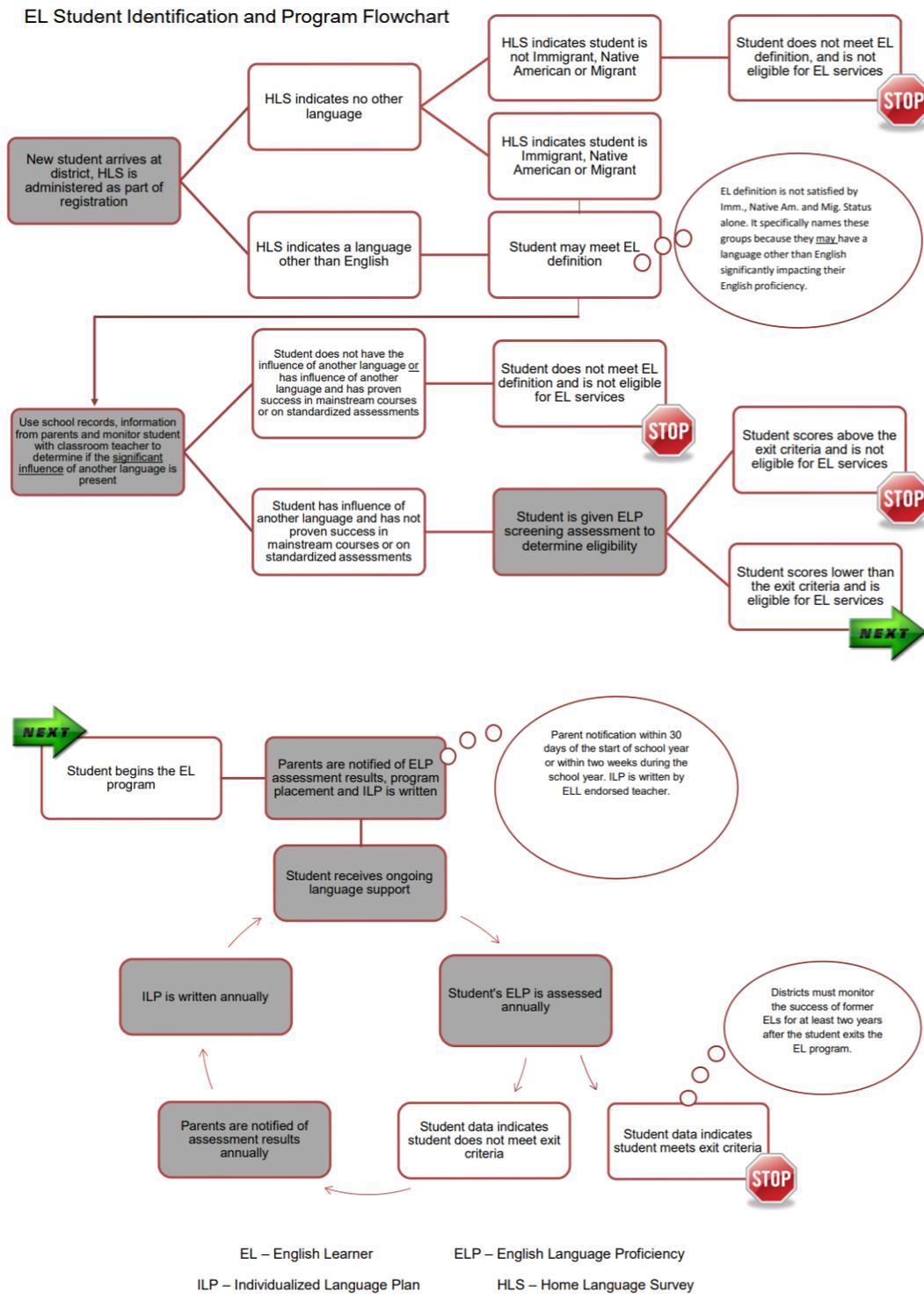
### **Language Proficiency Levels**

The WIDA MODEL, Screener, and ACCESS tests are broken down by grade levels (K, 1-2, 3-5, 6-8, 9-12). Within each grade level cluster, the ELP standards are embedded, and of the four language domains are represented separately, and each academic language area in language arts, mathematics, science, and social studies are also intertwined within the testing questions.

Below in Figure 1 is a summary of the identification and placement process for EL students as explained above in a flow chart that is used in our district.

Figure 1

District EL Student Identification and Program Flowchart



Note: Unpublished document developed by personnel at the district level.

**Content**

Echevarria et al. (2004) indicate that although there may be a language other than English spoken in the students' homes, there are two other scenarios that affect the instruction in school. On one hand, there is a group of students within this range who will be literate in their home language and will rely greatly on their background knowledge to assist in the learning of not only the subject matter but also the academic and social vocabulary that goes with it in the English language. On the other hand, there will be students who are not literate in any language. These students will have nothing to grasp onto when they are being taught a new subject matter in a new language in written materials because their background knowledge does not include anything similar to the new material. "Language is the primary vehicle for learning, instruction, and overall intellectual development. It is not only a means for communication of information, it is also a vehicle for helping learners broaden and deepen their understanding of important ideas," said Kersaint, Thompson, & Petkova (2009, p. 46).

There is a common misconception that math is a universal language and students who have had prior educational experience that included mathematical instruction should excel in math, regardless of their first language. The truth is that making sure all students understand math vocabulary and have ample opportunities to use it are very important. Solving word problems, following instructions, understanding and using mathematical vocabulary correctly are all skills that require a language proficiency that sometimes exceeds our expectations. Imagine a student who is placed in a mainstream mathematics classroom setting who either has a low reading proficiency or has not been exposed to mathematical vocabulary. The math worksheet says, "Look at the table." And your student starts to look around the room for a table to sit at, not at the data table on their worksheet in front of them. Imagine another student in an algebra or

geometry classroom is given a worksheet that says, “Find  $x$ .” The student might circle the letter  $x$  instead of using algebraic skill to solve for the missing number  $x$ . Now imagine all of the other academic vocabulary terms that might be used in mathematics that have more than one meaning or the student has never been exposed to before. The likelihood of their success is severely hindered by their lack of vocabulary. (Robertson, 2016).

English Learner students who enter the United States have many different backgrounds in any given classroom, both linguistically and culturally. Many of these students will have languages other than English spoken in their homes, often with parents who have no English at all. There will also be students who enter the classroom who have had little to no interaction with peers and are lacking immensely in social awareness and belonging. There are English learner students who have had no schooling prior to entering the United States; possibly due to lack of resources, war in their home country, or safety and health issues. There will also be students who enter the school systems with an abundance of subject matter knowledge (Echevarria et al., 2004).

### **Culture**

The amount of background knowledge and familiarity of the American culture, or lack thereof, that the EL students enter the classroom with can also play a huge role in the miscommunication that they have in the classrooms, with the materials, with peers, and with teachers. Many of the materials used throughout the United States appear to be written with the assumption that all students being exposed to them have been raised in the United States culture. Teachers’ ability to recognize cultural bias in their own materials and modify their lessons in such a way that it is understood by all students is imperative if the expectation is to reach the students both socially and academically.

There are several students who come to the United States with very strong academic backgrounds prior to entering our schools. Some of these students will have completed grade levels equivalent to grades in the United States' school system in subjects such as math and science, explain Echevarria et al. (2004). Slavit & Ernst-Slavit (2007) add that if the new EL students have previous content knowledge, the transition to a similar academic setting in a second language will most likely be much easier than for an EL student who has not had any schooling available to them for reasons mentioned earlier. The language behind mathematics is one of the hardest things to grasp when it comes to academic achievement. Students who come to school in the United States can be very talented at mathematics, but if they have a low English language proficiency, they will likely fail in the mainstream classroom. Mastery of the English language, academically and socially, often needs to come prior to mastery of mathematics. Slavit & Ernst-Slavit (2007) state that talking math ("using specific academic language needed to learn and express mathematical knowledge" (p. 4) has many challenges with its use of specific vocabulary. There are numerous words and phrases in mathematics that mean something completely different in everyday contexts, such as simple words like table, even, yard, key, volume, right as in right triangle, and more complex words like circular, rational, and orientation. There are also several types of vocabulary common in the mathematics classroom, such as high-frequency vocabulary (small, orange, clock), general vocabulary (combine, describe, consequently), specialized vocabulary (number, angle, equation, average), and technical vocabulary (perfect numbers, supplementary angles, quadratic equations, cosine, mode). All students, EL students included, are required to draw from all four of the vocabulary types when participating in mathematical conversations.

Word problems can be extremely difficult to comprehend with a lack of mathematical vocabulary for EL students. “Written word problems present a unique challenge to EL students as well as the teachers. In *Reading and Understanding Written Math Problems*, Brenda Krick-Morales writes, "Word problems in mathematics often pose a challenge because they require that students read and comprehend the text of the problem, identify the question that needs to be answered, and create and solve a numerical equation — ELs who have had formal education in their home countries generally do not have mathematical difficulties. Therefore, their struggles begin when they encounter word problems in a second language that they have not yet mastered" (In Bernardo, 2005) Robertson explains (2016).

EL students who are new to reading the English language have a difficulty with comprehending complex sentences and paragraphs. In mathematics, these students can be faced with large paragraphs in the form of a question (a math word problem) filled with words they may not understand and possibly a cultural bias they may not catch on to. For example, there may be a word problem that involves a simple recipe of a dish made for the typical American child, surely everyone will recognize like ice cream or a donut. However, EL students who are new to the United States may have never heard of, seen, or eaten these foods, and are not expected to get hung up on these simple words before understanding what mathematically they are to do with the word problem.

### **Stages of Development**

It is important that teachers show an unlimited amount of patience with EL students as they stumble through the development stages of learning a new language. EL students will start speaking single words and move into short utterances or the main subject-verb chunk of a sentence without a complete idea. They will slowly move to full sentences that may contain

several grammatical errors before speaking more like their native peers. It is also said that an EL student will master the language domain of speaking before reading or writing, which is also extremely important to remember when considering modifications inside and outside the EL classrooms (Kersaint et al., 2009).

Teaching content to English Learners (ELs) comes with many factors that need careful consideration. Teaching any student at any grade level is a difficult task with lack of communication. In order to communicate effectively, teachers need to have knowledge of all factors such as native language background, cultural background, literacy levels, and academic achievements of each student. Echevarria, Vogt, and Short (2004) explain that there is a wide range of English learner students entering the United States and teachers need to be aware of their vast differences. These students enter the school system with different levels of language proficiencies, knowledge of subject matter, and many other variables. The likelihood of their success depends largely on the amount of additional supports and different pathways they will receive in their new educational environment.

### **Chapter 4: Solutions for ELs**

To transform the way ELs learn with effective strategies, SLIFE and language barriers need to be addressed in the curriculum and in the classroom. The language of mathematics includes symbols, conventions, terms like coefficient, acute, ray, and other vocabulary terms that have specific meanings in math. The language of mathematics holds linguistic challenges that even native English-speaking students often struggle with. Math is taught through language, hence teachers of math are also teachers of language. The content simply cannot be taught without both. More times than not, when a student is struggling with achievement levels in mathematics, they likely are struggling with the linguistic complexity of the curriculum (Fredrickson, 2019).

Sheltered instruction is one major way that teachers have been able to make content matter more accessible to English Learners. Echevarria et al. (2004) state that “sheltered instruction (SI) integrates language and content and infuses sociocultural awareness while scaffolding instruction for students learning English. Sheltered instruction provides a bridge to the mainstream classroom as the teacher in the SI classroom has an understanding of the capacity at which the student can understand English and tailor the instruction to their levels of understanding. (p. 16)” Modifying language in the classroom is one main way to make content more accessible to EL students.

There are several ways of modifying content using a variety of teaching strategies to boost understanding of grade appropriate and age-appropriate subject matter. Rosen and Sasser (1997) suggest that sheltered instruction lessons provide cognitive and linguistic scaffolds for

English learners through techniques such as modeling, demonstration, and especially interaction with the teacher and peers, using English language skills to demonstrate and communicate about their comprehension of the content materials. The Sheltered Instruction Observation Protocol (SIOP) method follows this vision and teaches the importance of language acquisition, building background knowledge, increasing student language production, and explicitly teaching academic language. Using these techniques for more effective instruction, students should be able to better understand content and work together to find creative ways to learn. The SIOP model also aids in prompting discussion of mathematical concepts and how to use instructional supports the teacher has given the EL students as well as making these students more comfortable with mathematics and asking questions (Robertson, 2016). As an additional benefit, the SIOP model for lesson planning and teaching can assist teachers in all content areas, whether they have strictly EL students or a mixture of mainstream students and EL students.

Everyone can benefit from the variations of teaching styles while boosting language and content learning goals. The SIOP model integrates structures recommended for high-quality instruction for all students like reading comprehension strategies and cooperative learning, while specifically adding features for EL students such as inclusion of language objectives in each lesson, oral language practice, and development of background knowledge and academic vocabulary (Short, Fidelman, & Louguit, 2012). Although sheltered instruction is preferred for teaching students with low proficiency, this should not imply that EL students should have no instruction of academic content with their native English-speaking peers. It is quite the opposite. In most cases, EL students are placed in mainstream classes due to lack of professionals highly trained in both subject matter and ESL instruction, but studies have shown that immersion into mainstream classrooms has a high positive impact on EL students learning a second language.

Hearing more than one person (their teacher) speak in the language they are learning helps not only the social aspect of learning the second language, but their own speaking abilities are increased as well. However good for an EL student to be immersed in a mainstream classroom for either reason, it is also extremely important that the mainstream teachers modify their curriculum to reach levels of understanding for the EL student just as they would their struggling mainstream student (Kersaint et al., 2009).

To better serve the EL population in the district where I work, student data must be collected and reviewed to their English language proficiency and their mathematics proficiency. This data will assist in proper course placement for each individual student with their own unique scores and backgrounds.

The students will first be selected by their English proficiency levels and their mathematic ability levels and placed in the EL mathematics classroom accordingly. The tests that will be used for this study to determine the students' English proficiency are the WIDA Screener and the WIDA ACCESS 2.0. The tests that will be used for this study to determine the students' mathematics proficiency will be the NWEA MAP test, the Scholastic MI test, and the EL Math Placement Test.

The English proficiency tests given are called the WIDA Screener which is an English proficiency assessment given to incoming students, new to America or new to the district with no WIDA scores from their previous schooling, in grades 1-12 to assist educators with the identification of students as English Learners (ELs). The purpose of the assessment is to help educators make decisions as to whether the student is a candidate for English language support services. This test focuses on the four domains of language; reading, writing, listening, and speaking, and helps to show educators how proficient the individual student is in each of these

domains. The WIDA ACCESS 2.0 test is a larger scale proficiency assessment given to every identified EL student in the district once per year to monitor progress of the four language domains.

The math proficiency will be tested by three different tests, two of which are given by the district, and one specifically from the EL teachers. The first district test is the NWEA MAP (Northwest Evaluation Association Measures of Academic Progress) test and is given to every student entering the district. The MAP test measures a students' personal academic progress which is then reported as a number based on average grade level proficiency. Validity and reliability are good. The NWEA assessments are guided by the Standards for Educational and Psychological Testing. The NWEA Research tea regularly conducts a variety of studies and analyses such as: pool depth analysis, test validation, comparability studies, and Differential Item Functioning Monitoring item quality to ensure that functioning remains constant across subgroups of students when ability is controlled.

The second district test is the Scholastic Math Inventory (MI) test, and this is also given to every student entering the district. The MI test is assessed through a computer based intervention program called Math 180. Math 180 is intended for the use of grades 5 through high school and is meant to rebuild the foundational skills that struggling students need to prepare for algebra readiness. The MI is an adaptive test given to students K-12 which focuses on number identification, fact fluency, and attending to math precision. Validity and reliability are good. Scholastic uses four sources of measurement error to examine the MI test; marginal reliability, consistency, quantile measure, and standard error or measurement. There are several studies and findings toward validity of the MI test in areas such as content-description validity, criterion-prediction validity, and construct-identification validity.

Finally, we give the EL students a math placement test developed by the districts' EL teachers. This test eliminates much of the language in computational questions, but also tests language in areas that might lead to further placement. The validity and reliability have not been tested in any studies thus far, as only the EL teachers within the district use this test. It is not used for anything other than course placement within the school, and it is coupled with the consideration of the scores in the two other tests mentioned. All three of the math placement tests have multiple choice questions and open-ended items.

Oftentimes, the fifth or sixth grade teachers will recommend a student be placed in an intervention math class if they are struggling in the mainstream setting, and prior to their schedule switch or placement, it is my job as the EL math teacher to determine what the student is struggling with. If they are proficient in English and struggling with mathematics concepts, it is likely they do just need a mathematics intervention. But, most of the time, it is not that the student does not understand the mathematics content, but the academic language behind it. To determine this, I look at all the tests mentioned earlier prior to placing the student in an appropriate class. By holding a one on one conversation with the fifth, sixth, seventh, and eighth grade teachers, I can get a better understanding of what topics and content the students struggle with the most and try to implement more of that into my EL mathematics class.

### **SLIFE Solutions**

There is also a need to condense anywhere from 1 to 8 years of math content into just one or two short years in the EL mathematics class. Communicating with the teachers in the middle school will help me understand the standards that are essential to help them be successful in their future math classes. There is also a need to research standards and materials used in the district prior to sixth grade, which I have done and shown previously in the tables from Chapter 2.

## Language Solutions

The research shows that there are several strategies for teaching mathematical vocabulary that will support EL students as they learn to talk math. Vocabulary should not be taught as a separate entity, but as part of a lesson. Visual aids, hands on objects, and manipulatives that go along with the vocabulary can assist the students' memory and support conversations. It has also been recommended by prior research that less than 12 words per lesson will optimize retention of vocabulary as well as chunking material into smaller digestible bites rather than all at once (Fathman, 2007). As EL students are learning English and new mathematical concepts, it is often difficult to decipher which new words are key vocabulary or just unfamiliar sentence vocabulary, so teachers are advised to identify or highlight these key terms within lessons to assist the student's understanding of the main objectives. Furthermore, repetition and asking questions of understanding too frequently is preferred to not asking enough (Slavit & Ernst-Slavit, 2007).

It is also important to demonstrate and to make a point that the new vocabulary EL students are learning can have multiple meanings. Some students may wonder why that new word sounds so familiar, and it is at this point that the teacher should recognize the EL students may know this word already, but not yet in the mathematical context that pertains to the lesson. Giving students several opportunities to communicate with each other is essential to their increasing proficiency in both the English language and in mathematics. Some ways to increase student interaction might be to have students read math sentences aloud, translate symbols into words and write the sentence out, creating sentence frames to be used in each lesson, having students share problem-solving strategies, allowing a discussion in the classroom about what students are thinking about, and incorporating writing activities like a math journal (Robertson, 2016). Discussion within the classroom led by the teacher will play a huge role in what the

students are learning or taking away from each lesson. Asking the students to consider why or how will force students to reflect on their own thinking processes. Typically, student participation in the classroom in both discussion and following mathematical processes led through explicit instruction will allow the EL student to learn mathematics while simultaneously learning English. (Kersaint et al., 2009).

### **Chapter 5: Sheltered Math Curriculum for Middle Level ELS**

The trouble that many teachers have when designing a curriculum is deciding what information in the curriculum is more important than the rest, what is worth leaving out or skipping, and who decides these things? A colleague of mine who has also been implementing my adapted curriculum for teaching mathematics to EL students sat down with me over the summer and attempted to rewrite some sequencing of curriculum instruction for the EL students with the lowest mathematics and English language proficiencies. This is the exact dilemma we encountered. What do we include in our curriculum from the K-5 scope? What do we leave out? How is it up to us to decide this alone? Unfortunately for us and for the rest of the world, this is an ongoing internal struggle that will have to remain as such. We decided to use forward thinking when making our decisions. What will impact their future studies the most, and which topics will have less of an impact if we didn't cover certain matters? In doing so, we looked ahead to future scopes of seventh grade, eighth grade, and even high school mathematics courses.

Along with the K-5 curriculum, we also had to figure out a way to integrate the 6<sup>th</sup> grade and some of the 7<sup>th</sup> grade mathematics curriculum into the scope for our EL students, for those who might be near proficiency of both math and English. If a student is with us in 6<sup>th</sup> grade, they may be able to enter 7<sup>th</sup> grade math with their mainstream peers the following year, but with that, they would need the proper preparation. See Appendix B for a list of grade level goals the district uses for 6<sup>th</sup> grade and 7<sup>th</sup> grade math in the mainstream classrooms.

In creating curriculum for the sheltered mathematics classroom for middle level EL students, there were a few main concepts I centered the content around. First, I wanted to be certain that each head topic covered in the elementary mainstream math curriculum was also covered in this course: numbers, operations, data, measurement, geometry, patterns, and algebra concepts. In order to truly serve as an intervention in simplest of terms, I had to ensure that no concept was missed. The second main emphasis was how to guarantee that language was built into every section, be it spelling, vocabulary, pronunciation, root words, affixes common in math, terms with multiple meanings, and even basic grammar. Therefore, I have designed a two-part curriculum that should be completed as a series. Each level is unique, but the second builds on the first.

The goal when creating this curriculum was to combine six to seven years of schooling in the subject of mathematics into two, possibly three consecutive years of crash course curriculum. Level one starts with the basics of numbers: what they look like, how to spell them, the idea of place value, how to properly combine them and other words in the English language that translate to the meaning of addition and subtraction. Level one also involves topics we take for granted when we send our children to school like reading a clock or a calendar or working with coins and dollar bills. You'll also note below that the multiplication tables are broken up into separate units. It takes mainstream students several years to master basic math facts, but if students miss those important first few years, it is unfair to assume they will catch on in the blink of an eye. This also shows in my curriculum where certain topics are repeated several times, but built upon little by little. Repetition coupled with continued use and revisitation helps continued education on the subject instead of a "use it then lose it" type of learning. See Table 1 and Table

2 for the scope and sequence written for the two levels of EL Mathematics Curriculum for Middle Level ELs that I have developed.

Table 1: *EL Math 1 Scope & Sequence*

<b>EL Math 1 – Concept Sequence</b>
<p><u>Unit 1: Numbers 1-20, Addition</u></p> <ul style="list-style-type: none"> <li>● Read numbers to 20, Write numbers to 20 (in digits and words), Place value (ones and tens place), Ordering/Comparing numbers up to 20 (use <math>&gt;</math>, <math>&lt;</math>, <math>=</math>) <ul style="list-style-type: none"> <li>○ Say/Pronounce Numbers (thirty vs thirteen)</li> <li>○ Listen to numbers / understand the numbers when they are spoken</li> </ul> </li> <li>● Number lines to 20 (whole numbers) left to right</li> <li>● Counting by 1s, 2s, and 5s (forward and backward)</li> <li>● Number patterns (forward and backward)</li> <li>● Adding single digits</li> <li>● Addition vocabulary, symbolic and everyday terminology <ul style="list-style-type: none"> <li>○ Everyday vocabulary (ex. “and”, “combined”, “together”, “how many”, “join”, “total”)</li> <li>○ Academic vocabulary (ex. “plus”, “sum”, “add”)</li> </ul> </li> </ul>
<p><u>Unit 2: Numbers to 100, Rounding, Addition and Subtraction, Coin Intro</u></p> <ul style="list-style-type: none"> <li>● Read numbers to 100, Write numbers to 100 (in digits and words), Place value (to hundreds place), Ordering/Comparing numbers up to 100 (use <math>&gt;</math>, <math>&lt;</math>, <math>=</math>) <ul style="list-style-type: none"> <li>○ Say/Pronounce Numbers</li> <li>○ Listen to numbers / understand the numbers when they are spoken</li> </ul> </li> <li>● Number lines to 100 (whole numbers) left to right</li> <li>● Counting by 10s, 20s (forward and backward)</li> <li>● Number patterns (forward and backward); Explaining a pattern</li> <li>● Round to tens place, Real World Problems with Estimation</li> <li>● Adding numbers up to 100 <ul style="list-style-type: none"> <li>○ Adding without carrying, Adding with carrying</li> </ul> </li> <li>● Subtract numbers 1-20; Subtracting without borrowing</li> <li>● Subtraction vocabulary, symbolic and everyday terminology <ul style="list-style-type: none"> <li>○ Everyday vocabulary (ex. “less”, “taken away”, “reduce”, “decrease”, “left over”, “remains”, “remove”, “deduct”)</li> <li>○ Academic vocabulary (ex. “minus”, “difference”, “subtract”)</li> </ul> </li> <li>● Coin (American currency) names and values (Penny, Nickel, Dime, Quarter)</li> <li>● Adding coins up to \$1.00 by word name only and by picture only <ul style="list-style-type: none"> <li>○ Real world application using money, items, numbers of coins</li> </ul> </li> </ul>
<p><u>Unit 3: Numbers to 1,000, Rounding, Add/Subtract, Coins and Bills Computation</u></p> <ul style="list-style-type: none"> <li>● Read numbers to 1,000, Write numbers to 1,000 (in digits and words), Place value (to one thousands place), Ordering/Comparing numbers up to 1,000 (use <math>&gt;</math>, <math>&lt;</math>, <math>=</math>)</li> </ul>

- Say/Pronounce Numbers
- Listen to numbers / understand the numbers when they are spoken
- Correct placement of commas and how to “pause” at the comma when reading numbers; How to notice a comma in number words or a pause in number words and recognize the place value when hearing or reading numbers in thousands
- Number lines to 1,000 (whole numbers) left to right
- Counting by 25s, 50s, 100s (forward and backward)
- Number patterns (forward and backward), Explaining a pattern
- Round to hundreds place, Real World Application using Estimation
- Adding numbers up to 1,000
- Subtract numbers to 100
  - Subtract with borrowing
- Number sentences, Reading, Writing, Listening, Speaking equations and expressions with addition and subtraction; Recognizing the vocabulary for each
- American currency dollar bills and coins – adding money denominations, real world application using money

#### Unit 4: Numbers to 100,000, Even/Odd, Exchanging Money

- Read numbers to 100,000, Write numbers to 100,000 (in digits and words), Place value (to hundred thousands place), Ordering/Comparing numbers up to 100,000 (use  $>$ ,  $<$ ,  $=$ )
  - Say/Pronounce Numbers
  - Listen to numbers / understand the numbers when they are spoken
  - Correct placement of commas and how to “pause” at the comma when reading numbers; How to notice a comma in number words or a pause in number words and recognize the place value when hearing or reading numbers in hundred thousands
- Round to one thousands and ten thousands place, Real world application of estimation
- Multi-digit addition and subtraction problems
- Recognizing Even / Odd Numbers, Even / Odd meaning
- Exchanging American Currency + Different combinations of coins; both picture coins and coins names

#### Unit 5: Clocks & Calendar

- Clocks/Telling time, Clocks/Showing time
  - Reading a clock properly; vocabulary, order, clockwise, counter clockwise
- Read/Write time with more traditional fraction terminology: half past, quarter after, quarter past, quarter to (recognizing these terms and understanding what it means when referring to time)
- Ordinal numbers (first, second, third, etc.) and how to apply them to real world situations
- Months of the year (say, pronounce, spell, write in order), Days of the week (say, pronounce, spell, write in order)
- Elapsed time computation (within 2 hours) and real-world application of elapsed time. Recognizing vocabulary for elapsed time (hours or minutes later, time past, ago, etc.)

Unit 6: Tables/Graphs

- Addition/Subtraction Synonym words – Everyday and Academic Vocabulary
  - More than, Less than, Exceeded by, Increase/Decrease
- Introduction to Reading tables/graphs
  - Tables + Tally Charts, Single Bar Graphs, Single Line Graphs, Circle Graphs / Pie Charts – quantities, no percents
- Introduction to Creating tables/graphs
  - Tables + Tally Charts, Single Bar Graphs, Single Line Graphs, Circle Graphs / Pie Charts – quantities, no percents

Unit 7: Multiplication + Geometry Intro

- Multiplication Facts: 0, 1, 2, 5, 10 tables
- Multiplication vocabulary, symbolic and everyday terminology
  - Everyday vocabulary (ex. “each”, “double”, “twice”, “area”, “by”, “groups of”, “per”)
  - Academic vocabulary (ex. “times”, “product”, “of”)
- Identify 2D Shapes Intro: spelling shapes, recognizing vocabulary, matching shape name to picture
  - Attributes of each shape, meaning of sides, edges, corners
- Continue Shapes Patterns and Letter Patterns; explaining patterns
- Perimeter vocabulary, Finding Perimeter mathematically, Real-world application of perimeter
  - Everyday vocabulary ex. border, boundary, around, outline, edges

Unit 8: Multiplication, Division Intro, Geometry

- Multiplication Facts: 3, 4, 6 tables
- Symmetry, Congruent figures
- Division Facts: 0, 1, 2, 5, 10 tables
- Division vocabulary, symbolic and everyday terminology
  - Everyday vocabulary (ex. “each”, “half”, “goes into”, “equal parts”, “evenly”, “split”)
  - Academic vocabulary (ex. “divide”, “quotient”, “dividend”, “divisor”, “remainder”)

Unit 9: Multiplication/Division + Geometry Vocab

- Multiplication Facts: 7, 8, 9 tables
- Direction/Position vocabulary
  - Left/Right, Top/Bottom, Above/Below, In front/Behind
- Point/Line/Segment vocabulary
- Identify, name, pronounce, spell simple 3D shapes (prism, pyramid, cube, cylinder)
- Division 3, 4, 6

Unit 10: Multiplication

- 2 x 1 and 3 x 1 without carrying, 2 x 1 with carrying
- Perimeter/Area with 2x1 addition and multiplication
- Area vocabulary, Finding area mathematically, Real-world application of area

- Everyday vocabulary ex. space, within, field, zone, stretch, surface
- Division Facts: 7, 8, 9 tables; review all division tables

#### Unit 11: Fractions

- Reading/naming fractions/mixed numbers
  - Applying ordinal numbers spelling, pronunciation to fraction words
  - Applying speech pause, separation, connector words in speech to fraction bar
    - Out of, to, over
  - Real world application to fractions
- Equivalent fractions, Comparing fractions
  - Real world application to equivalent fractions
- Fractions on a number line
- American Rulers (parts of 1 whole) – inches, Draw segments – inches

#### Unit 12: Decimals

- Reading/naming decimals, Place value to ten-thousandths, Decimals tenths and hundredths on number line
- Rulers – cm and mm (base 10 system align with decimals), Draw segments cm and mm
- (Optional) Convert fractions and decimals

#### Unit 13: Division

- 2 digit divided by 1 digit without remainders, 2 digit divided by 1 digit with remainders
- Equations with missing number blanks (no variables)
- Translating equations
- Number sentences Reading, Writing, Listening, and Speaking with multiplication and division facts

#### Unit 14: Probability

- Introduction to the concept: coins, cards, bags with colored beans, balls, candy, etc.

There is an obvious overlap between the Level 1 sequence and the Level 2 sequence. This is intentional, as the hope is to serve these students in two to three consecutive years. The second year, the student repeats many of the concepts that were learned throughout the entire first year of EL math curriculum, at a faster more rigorous pace than the previous year and builds up from there. You may notice this specifically in Level 2, Units 1A and 1B, where the topics being covered in these two units are skills the students spent the first half of the previous year

mastering. All topics covered in Level 2 concept sequence involve the same rigor of vocabulary and language learning intensity noted in Level 1.

Table 2: *EL Math 2 Scope & Sequence*

<b>EL Math 2 – Concept Sequence</b>	
<u>Unit 1A: Numbers</u>	
<ul style="list-style-type: none"> <li>● Read &amp; Write numbers to billions (in digits and words), Place value to billions, Expanded Form, Ordering/Comparing large numbers (use &gt;, &lt;, =)</li> <li>● Number patterns (forward and backward)</li> <li>● Number lines (whole numbers)</li> <li>● Rounding</li> </ul>	
<u>Unit 1B: Review Multiplication &amp; Division Facts</u>	
<ul style="list-style-type: none"> <li>● Repeated Addition + Multiplication Visual Groups (for lower classes - intro to times)</li> <li>● Multiplication               <ul style="list-style-type: none"> <li>○ Facts 0-10, Multiplication up to 3 digit x 2 digit, Multiplication Word Problems</li> </ul> </li> <li>● Division Facts 0-10, Long Division - No Remainders, Division Word Problems</li> <li>● All Operation Vocabulary</li> </ul>	
<u>Unit 1C: Multiplication &amp; Division Extras</u>	
<ul style="list-style-type: none"> <li>● Divisibility, Finding Factors</li> <li>● Composite / Prime (Must master long division first)               <ul style="list-style-type: none"> <li>○ How to determine P/C</li> <li>○ +Vocabulary of each</li> </ul> </li> <li>● Finding GCFs: Factor Method + Tree Method</li> <li>● Distributive Property Intro               <ul style="list-style-type: none"> <li>○ Distributive property with GCFs                   <ul style="list-style-type: none"> <li>■ GCF of 12, 18 = 6, Use Distributive property/GCF to add:</li> <li>■ <math>(12 + 18) = (6 \times 2) + (6 \times 3) = 6(2+3) = 6(5) = 30</math></li> </ul> </li> <li>○ Distributive property - Multiplication with Expanded Form:                   <ul style="list-style-type: none"> <li>■ <math>17 \times 8 = 8(10+7) = (8 \times 10) + (8 \times 7) = 80 + 56 = 136</math></li> </ul> </li> </ul> </li> <li>● Division with Remainders, Division with 2 digit divisors (with table)</li> </ul>	
<u>Unit 2A: Fraction Introduction</u>	
<ul style="list-style-type: none"> <li>● Reading/naming fractions, Writing fractions in words</li> <li>● Equivalent Fractions</li> <li>● Simplifying Fractions / Higher Denominators: Common factors, GCF</li> <li>● Like/Unlike Fractions, Compare/Order Fractions, LCM</li> </ul>	
<u>Unit 2B: Decimal Introduction</u>	
<ul style="list-style-type: none"> <li>● Modeling tenths with fractions circles THEN with base-10 blocks</li> </ul>	

- Tenths and hundredths with decimals
- Compare/Order Decimals to hundredths
- Using rulers to model tenths of a centimeter (millimeters)

#### Unit 2C: Mixed, Improper, Whole Introduction / Operations

- Whole fractions, mixed numbers, and improper fractions
  - Reading/Naming, Writing in words
  - Converting mixed → improper and improper → mixed
  - Compare/Order
- Fractions, Mixed, Improper on a Number Line

#### Unit 3A: Decimal Operations

- Decimal place value to ten-thousandths, Reading/naming decimals to ten-thousandths, Writing decimals in words, Compare/Order Decimals
- Decimals on a number line
- Round/estimate decimals
- Adding and Subtracting decimals, Multiply and Divide decimals
- Long division decimals (with divisors in tenths, no 2-digit divisors)
  - Include Long Division Decimals + Repeating Decimals

#### Unit 3B: Fraction Operations

- Multiples, LCM, Prime Factorization - LCD
- Add/Subtract Fractions & Mixed Numbers ( + Review Simplify Fractions)
- Multiply/Divide Fractions & Mixed Numbers ( + Review Simplify Fractions)

#### Unit 4: Tables/Graphs + Ratios/Rates

- Reading tables/graphs
  - Tables & Tally Charts, Bar Graphs + Double bar graphs, Line Graphs + Double line graphs, Circle Graphs / Pie Charts
- Creating tables/graphs
  - Tables & Tally Charts, Bar Graphs + Double bar graphs, Line Graphs + Double line graphs, Circle Graphs / Pie Charts
- Intro to Ratios/Unit Rates + Proportions
- Fractions/Decimals/Percents (understand connection & convert)
- 1st Quadrant Graphing

#### Unit 5A: Algebra Introduction

- Combining like terms, Writing simple expressions: Expression vocab – variable, operation, solution, inverse operation
- Distributive property, Exponents / Powers, Order of Operations
- Number Properties

#### Unit 5B: Algebra Intro 2

- Solve simple expressions with substitution
- Using Tables for Functions – follow patterns; fill in blanks, Input/Output vocabulary
- Solving one-step equations: With addition and subtraction, Using “fill in the blank” before using variables, With multiplication and division
- Translate expressions & equations (positive numbers)

#### Unit 6: Integers

- Integers Introduction, Integers on a number line, Integer vocabulary in everyday life
- Coordinate plane - all quadrants
- Absolute Value & Opposites
- Integer Operations: Add/Subtract, Multiply/Divide

#### Unit 7: Equations & Inequalities

- Solving one-step equations with integers: With addition and subtraction, Using “fill in the blank” before using variables, With multiplication and division
- Translate expressions & equations with integers
- Introduce inequalities, Inequalities on a number line, Solving inequalities with addition and subtraction

#### Unit 8A: Geometry

- Review basic shapes
- Geometry Vocab: Point, Line, Segment
- Ruler measurement: (cm and mm and inches to  $\frac{1}{4}$  inch)
- Classify Angles: Draw and measure with protractor, Parallel/Perpendicular
- Classify Triangles: Use protractors to classify by angles, Use rulers to classify by sides
- Introduce and classify polygons and n-gons: Classify Quadrilaterals
- Extra Vocab: Congruent / Similar, Transformations

#### Unit 8B: Geometry

- Perimeter / Area, Identify 3D shapes (Faces, Edges, Vertices vocab)
- Volume / (Surface area if time)

#### Unit 9: Algebra Concepts 2

- Two-step equations
- Mean, median, mode, range
- Line plot, Dot plots, Histograms, Box Plots
- Probability Concepts

## **Chapter 6: Conclusion**

The purpose of this project was to understand and interpret challenges EL students face in the mainstream mathematics classroom. Language, content, culture, academic vocabulary, and SLIFE are major hurdles that ELs encounter, and the research done in this project provides solutions to them by diving into language proficiency and math curriculum needed to succeed in the middle school setting and beyond by creating a new mathematics curriculum for sheltered instruction of EL mathematics at the middle school level.

The research of this project summarizes the federal regulations as well as court cases and major rulings involving student rights, and legislation in North Dakota which requires all school districts to develop an EL program plan which identifies each EL student and their language proficiency, and places ELs in academic mainstream and EL courses accordingly with a language plan fit to their specific needs followed by all instructional staff. The students are monitored, assessed annually for proficiency, and exited from the EL program when their scores reach proficiency level. EL students struggle with the mainstream mathematics curriculum. Language and culture create a barrier that effects the ability to understand the academic language in math. My project presents the process and the final product used to appropriately place EL students in the sheltered instruction classes and the curriculum I created to meet the needs of the students with these struggles in the EL mathematics sheltered instruction classroom. My developed curriculum was created to meet the needs of the ELs in the middle school setting with focus on the language barriers and the educational gaps, vocabulary, and relevant content.

The curriculum I created has been implemented in my classroom for approximately six years. The students respond well to topics while applying math content and added vocabulary and language lessons to their learning. I have found that students will enter my classroom unenthused with the assumption that math is going to be hard, but through weeks and months of learning in my classroom, they show excitement in their knowledge and progress. While carefully adding a little content at a time and repeating many of the skills we learn early on throughout the year, the students show advancement and recall, and often surprise me with the strides they take from the beginning of the school year to the end. I have found success in sending students to mainstream after completing my Level 2 course with students expressing confidence in their abilities. However, I feel the need to keep students in my classroom for more than one year if they start my curriculum at Level 1, as there will still be educational gaps from the conclusion of Level 1 to any middle school grade level curriculum.

### **Next Steps**

My belief is that middle school EL students need material accessible to them in all proficiency levels in order to succeed. ELs should receive both English language support in their mathematics classroom and an adapted math content instruction differentiated to their level of need to achieve a higher success rate at algebra readiness than those who do not. There is a wealth of research already done and recommendations for teachers to modify materials for ELs. From my research findings thus far, the gap exists at the application level, not at the research level. After implementing this new curriculum, no data has been collected to prove the validity of its effectiveness. Feedback from teachers who have seen success in some of my students and feedback from teachers who serve these students in subsequent years of schooling has been positive.

The effectiveness of this curriculum could be addressed in the future with tangible data of a class of EL students receiving this modified math curriculum. The data could be collected and compared through the students' EL WIDA ACCESS scores and the annual state assessments all students are required to take. This data would show the increase of the English language proficiency as well as the improvement in their mathematical knowledge and ability to apply their new skills to grade level questions. It would also be beneficial to implement the curriculum in other buildings and other districts to investigate effectiveness of this curriculum outside my classroom.

### **Limitations**

However much progress I have made with ELs in my classroom, there is always going to be a call to modify and grow with the times. As different populations move into our school district, the needs of my students, as any district, change drastically from year to year. Curriculum is designed to meet the needs of the current classroom, and no one curriculum will ever fit the mold of every classroom. Additionally, the mainstream curriculum path is constantly changing and adapting to new materials and new sequences within each grade level course. The district is moving into a new grade reporting system called Evidence Based Reporting (EBR). This grading system focuses solely on the students' proficiency of content standards throughout the school year. The struggle I anticipate is that the curriculum I have built is not based on a single grade level set of standards but many intertwined. It will be challenging to fit this curriculum into the new methods in the direction the district headed. Nonetheless, my curriculum needs to reflect these changes so students moving on from EL Mathematics have a fighting chance to succeed in their future classes. It is my hope that the curriculum I have created continues to build on the district needs to appropriately serve the ELs in sheltered instruction.

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## Appendix A

Table 3

### *Kindergarten Mathematics Grade Level Goals*

Topic	Number and Numeration	Operations and Computation	Data and Chance
KINDERGARTEN	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Count by 1s, 2s, 5s, and 10s, to 100</li> <li>• Count objects</li> <li>• Place value to hundreds</li> <li>• Meanings and uses of fractions</li> <li>• Compare/Order up to 20</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Add/Subtract facts of single digit whole numbers</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Collect and organize data to make tally charts, tables, bar graphs</li> <li>• Use graphs to answer simple questions</li> <li>• Describe events using certain, possible, impossible, and other basic probability terms</li> </ul>
	<b>Measurement and Reference Frames</b>	<b>Geometry</b>	<b>Patterns, Functions, and Algebra</b>
	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Use nonstandard tools to estimate and compare weight and length</li> <li>• Identify money: different coins and dollars</li> <li>• Temperature vocab: cold, warm, hot</li> <li>• Identify a thermometer as a tool for measuring temperature</li> <li>• Time: day and week time periods; identify tools to measure time</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Identify and describe Plane and solid figures: circle, triangle, square, rectangle, sphere, cube</li> <li>• Identify shapes that have lines of symmetry</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Extend, describe, and create visual patterns</li> <li>• Find and use pattern rules</li> <li>• Read and write expressions and number sentences using symbols +, -, and =</li> </ul>

Table 4

*First Grade Mathematics Grade Level Goals*

Topic	Number and Numeration	Operations and Computation	Data and Chance
<i>1<sup>ST</sup> GRADE</i>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K +</li> <li>• Count back by 1s from any number &lt;100 without number grids, number lines, and calculators.</li> <li>• Count objects; estimate the number of objects in a collection</li> <li>• Place values up to 1000; identify places and the values of the digits in those places</li> <li>• Use manipulatives to model halves, thirds, and fourths</li> <li>• Use manipulatives to identify and model odd and even numbers</li> </ul> <p>Compare/Order #s up to 1000</p>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K +</li> <li>• Add/Subtract up to 20</li> <li>• Estimate and Compare simple addition/Subtraction</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K +</li> <li>• Line plots</li> <li>• Identify minimum and maximum of a data set</li> </ul>
	<p><b>Measurement and Reference Frames</b></p>	<p><b>Geometry</b></p>	<p><b>Patterns, Functions, and Algebra</b></p>
<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, + :</li> <li>• Measure length with standard measuring tools</li> <li>• Know and compare value of coins P, N, D, Q, and dollar bills</li> <li>• Make exchanges between coins</li> <li>• Read temperatures in Fahrenheit and Celsius</li> <li>• Use a calendar to identify days, weeks, months, dates</li> <li>• Tell and show time to nearest half and quarter hour on analog clock</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, + :</li> <li>• Identify and describe Plane and solid figures: Cylinder, rectangular prisms, pyramids, cones, and cubes</li> <li>• Complete line symmetric shapes or designs</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, + :</li> <li>• Read, write, explain expressions and number sentences using +, -, and =, and the symbols &gt; and &lt; with cues</li> <li>• Commutative and Associative properties of addition and Additive Identity</li> </ul>	

Table 5

*Second Grade Mathematics Grade Level Goals*

Topic	Number and Numeration	Operations and Computation	Data and Chance
2 <sup>ND</sup> GRADE	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, +</li> <li>• Count by 25s and 100s past 1000 and back by 10s and 100s from any #&lt;1000 with and without grids, number lines</li> <li>• Place values up to 10,000; identify places and the values of the digits in those places</li> <li>• R &amp; W money amounts in dollars and cents notation</li> <li>• Name fractions from given models</li> <li>• Recognize odd/even numbers</li> <li>• Use models to model equivalent names for <math>\frac{1}{2}</math></li> <li>• Compare/Order #s up to 10,000. Use area model to compare fractions</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, +</li> <li>• Use manipulatives, number grids, tally marks, mental arithmetic, paper and pencil to solve problems involving addition and subtraction up to 20</li> <li>• Calculate and compare values of coin and bill combinations</li> <li>• Reasonable estimation for addition and subtraction</li> <li>• Identify and describe change, comparison, and parts-and-total situations</li> <li>• Use repeated addition, arrays, and skip counting to model multiplication; use equal sharing and equal grouping to model division.</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, +</li> <li>• Mode, median of data set</li> </ul>
	<p><b>Measurement and Reference Frames</b></p>	<p><b>Geometry</b></p>	<p><b>Patterns, Functions, and Algebra</b></p>
	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, + :</li> <li>• Measure length to nearest inch and centimeter</li> <li>• Partition rectangles into unit squares and count unit squares to find area</li> <li>• Describe relationships between days in a week and hours in a day</li> <li>• Make exchanges between coins and bills</li> <li>• Tell and show time to the nearest five minutes on analog clock</li> <li>• tell and write time in digital notation</li> <li>•</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, + :</li> <li>• Draw line segments and identify parallel line segments</li> <li>• Identify and describe Plane and solid figures: Hexagons, trapezoids, rhombuses,</li> <li>• Create and complete two dimensional symmetric shapes or designs</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, + :</li> <li>• Describe rules for patterns and use them to solve problems; use words and symbols to describe and write rules for functions involving addition and subtraction</li> <li>• Solve number sentences involving addition and subtraction, write expressions and number sentences to model number stories</li> <li>• Describe commutative and associative properties of addition and additive identity</li> <li>• Apply properties to mental arithmetic problems</li> </ul>

Table 6

*Third Grade Mathematics Grade Level Goals*

Topic	Number and Numeration	Operations and Computation	Data and Chance
3 <sup>RD</sup> GRADE	<b>Detail:</b> <ul style="list-style-type: none"> <li>• K, 1, 2, +</li> <li>• R &amp; W #s up to 1,000,000, place value up to one million</li> <li>• Translate between whole numbers and decimals represented in words, in base-10 notation, and with manipulatives.</li> <li>• R, W and model fractions; solve problems involving fractional parts of a region or a collection</li> <li>• Find multiples of 2, 5, and 10</li> <li>• Use manipulatives to generate equivalent fractions</li> <li>• Compare/order #s up to 1,000,000; use manipulatives to order decimals through hundredths;</li> <li>• Use benchmark fractions to compare and order fractions.</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• K, 1, 2, +</li> <li>• use basic facts to compute fact extensions such as 80+70</li> <li>• Add/Subtract problems in a money context</li> <li>• Multiplication facts up to 10 x 10</li> <li>• Multiplication of 2 and 3 digit numbers by one digit number</li> <li>• Estimates for multiplication and division problems</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• K, 1, 2, +</li> <li>• Range of data set</li> <li>• Predict the outcomes of simple experiments and test the predictions using manipulatives;</li> <li>• express the probability of an event by using "_out of _" language.</li> </ul>
	<b>Measurement and Reference Frames</b>	<b>Geometry</b>	<b>Patterns, Functions, and Algebra</b>
	<b>Detail:</b> <ul style="list-style-type: none"> <li>• K, 1, 2, + :</li> <li>• Estimate length and without tools; measure length to nearest <math>\frac{1}{2}</math> inch and <math>\frac{1}{2}</math> centimeter; draw and describe angles as records of rotations</li> <li>• Describe and use strategies to measure the perimeter of polygons, find areas of rectangles</li> <li>• Describe relationships among inches, feet, and yards</li> <li>• Describe relationships between minutes in an hour, hours in a day, days in a week</li> <li>• Tell and show time to the nearest minute on an analog clock</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• K, 1, 2, + :</li> <li>• Identify and draw points, intersecting and parallel line segments and lines, rays, and right angles</li> <li>• Identify and describe Plane and solid figures: polygons</li> <li>• Use appropriate geometric terms to describe and identify such as face, edge, vertex, and base</li> <li>• Locate multiple lines of symmetry in a two-dimensional shape</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• K, 1, 2, + :</li> <li>• Describe and write rules for functions involving multiplication and use those rules to solve problems</li> <li>• Read, write, explain number sentences using x; solve number sentences; write expressions and number sentences to model number stories</li> <li>• Recognize that numeric expressions can have different values depending on the order in which operations are carried out; understand that grouping symbols can be used to affect the order in which operations are carried out</li> <li>• Describe and apply commutative and associative properties with multiplication; Multiplicative Identity, apply the distributive property over multiplication and addition</li> </ul>

Table 7

*Fourth Grade Mathematics Grade Level Goals*

Topic	Number and Numeration	Operations and Computation	Data and Chance
4 <sup>TH</sup> GRADE	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K,1, 2, 3, +</li> <li>• #s up to 1,000,000,000 and decimals through thousandths</li> <li>• Find multiples of #s &lt; 10</li> <li>• Prime and composite numbers</li> <li>• Find whole-number factors of numbers</li> <li>• Use numerical expressions involving one or more of the basic operations and grouping symbols to give equivalent names to whole numbers, fractions and decimals.</li> <li>• Use multiplication rule to find equivalent fractions</li> <li>• Rename fourths, fifths, tenths and hundredths as decimals and percents</li> <li>• Compare/Order #s up to 1,000,000,000; decimals through thousandths; integers between -100 and 0</li> <li>• Use analyses of numerators and denominators to compare and order fractions</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K,1, 2, 3, +</li> <li>• use basic facts to compute fact extensions such as 30*60.</li> <li>• Multiplication of multidigit numbers by 2 digit numbers</li> <li>• division of multidigit whole numbers by 1-digit whole numbers</li> <li>• Solve problems involving the addition and subtraction of fractions and mixed numbers</li> <li>• Estimating decimal addition and subtraction</li> <li>• Use area, and scaling to model multiplication and division.</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K,1, 2, 3, +</li> <li>• draw conclusions, and make predictions about the data set</li> <li>• Use more likely, equally likely, same chance, 50-50, less likely to compare events.</li> <li>• Summarize the results and use them to predict future events; express the probability of an event as a fraction.</li> </ul>
	<p><b>Measurement and Reference Frames</b></p>	<p><b>Geometry</b></p>	<p><b>Patterns, Functions, and Algebra</b></p>
	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, 2, 3, + :</li> <li>• Measure length to the nearest <math>\frac{1}{4}</math> inch and <math>\frac{1}{2}</math> centimeter; use tools to measure and draw angles</li> <li>• estimate size of angles without tools</li> <li>• Describe and use strategies to measure the perimeter and area of polygons, estimate the area of irregular shapes, and find the volume of rectangular prisms</li> <li>• Describe relationships among U.S. customary units of measure and among metric units of measure</li> <li>• Use ordered pairs of numbers to name, locate, and plot points in the first quadrant of a coordinate grid</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, 2, 3, + :</li> <li>• Describe points, intersecting and parallel line segments and lines, rays</li> <li>• Identify, draw, and describe right, acute, and obtuse angles</li> <li>• Compare and classify plane and solid figures; use appropriate geometric terms including congruent</li> <li>• Identify, describe, and sketch examples of reflections; identify and describe examples of translations and rotations</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, 2, 3, + :</li> <li>• Extend, describe, and create numeric patterns; describe rules for patterns and use them to solve problems; use words and symbols to describe and write rules for functions that involve the four basic arithmetic operations and use those rules to solve problems</li> <li>• Use conventional notation to write expressions and number sentences using the four basic arithmetic operations; determine whether number sentences are true or false; solve open sentences and explain the solutions; write expressions and number sentences to model number stories.</li> <li>• Evaluate numeric expressions containing grouping symbols; insert grouping symbols to make number sentences true.</li> <li>• Describe and apply the Distributive Property of Multiplication over Addition.</li> </ul>

Table 8

*Fifth Grade Mathematics Grade Level Goals*

Topic	Number and Numeration	Operations and Computation	Data and Chance
<b>5<sup>TH</sup> GRADE</b>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, 2, 3, 4, +</li> <li>• R &amp; W all whole numbers and decimals</li> <li>• All place values</li> <li>• Use expanded notation to represent whole numbers and decimals</li> <li>• Solve problems involving percents and discounts</li> <li>• Factor numbers, prime factorization</li> <li>• Use numerical expressions to represent percents</li> <li>• Simplify fractions; simplest form</li> <li>• Convert between fractions and mixed numbers</li> <li>• Convert between fractions, decimals and percents</li> <li>• Compare and order rational numbers</li> <li>• Compare mixed numbers</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, 2, 3, 4, +</li> <li>• Multiplication of whole numbers and decimals</li> <li>• Division of multi-digit whole numbers and decimals by whole numbers</li> <li>• Express remainders as whole numbers or fractions</li> <li>• Multiplication and division of fractions and mixed numbers</li> <li>• use ratios expressed as words, fractions, percents, and with colons</li> <li>• Solve problems involving ratios of parts of a set to the whole set</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, 2, 3, 4, +</li> <li>• Collect and organize data or use given data to create graphic displays with reasonable titles, labels, keys, and intervals.</li> <li>• Compare predictions based on theoretical probability with experimental results</li> <li>• Express the probability of an event as a fraction, decimal, or percent.</li> <li>•</li> </ul>
	<p><b>Measurement and Reference Frames</b></p>	<p><b>Geometry</b></p>	<p><b>Patterns, Functions, and Algebra</b></p>
	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, 2, 3, 4, + :</li> <li>• Measure length with tools to the nearest <math>\frac{1}{8}</math> inch and millimeter</li> <li>• estimate the measure of angles with and without tools</li> <li>• Use tools to draw angles of given measures</li> <li>• Describe and use strategies to find perimeter of polygons and area of circles</li> <li>• Choose and use appropriate methods, including formulas, to find areas of rectangles, parallelograms, and triangles, and the volume of a prism;</li> <li>• Define pi as the ratio of a circle's circumference to its diameter                             <ul style="list-style-type: none"> <li>• Use ordered pairs of numbers to name, locate, and plot points in all four quadrants of a coordinate grid</li> </ul> </li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, 2, 3, 4, + :</li> <li>• Identify, describe, compare, name, and draw right, acute, obtuse, straight, and reflex angles; determine angle measures in vertical and supplementary angles and by applying properties of sums of angle measures in triangles and quadrangles.</li> <li>• identify congruent figures and describe their properties.</li> <li>• Identify, describe, and sketch examples of reflections, translations, and rotations.</li> </ul>	<p><b>Detail:</b></p> <ul style="list-style-type: none"> <li>• K, 1, 2, 3, 4, + :</li> <li>• Represent functions using words, symbols, tables, and graphs and use those representations to solve problems.</li> <li>• Use a letter variable to write an open sentence to model a number story; use a pan-balance model to solve linear equations in one unknown.</li> <li>• Evaluate numeric expressions containing nested grouping symbols; insert nested grouping symbols to make number sentences true; describe and use the precedence of multiplication and division over addition and subtraction.</li> <li>• Describe and apply properties of arithmetic.</li> </ul>

## Appendix B

Table 9

*Sixth Grade Mathematics Grade Level Goals*

Unit 1: Decimals	Unit 2: Division of Fractions	Unit 3: Ratio Concepts and Reasoning	Unit 4: Expressions
<b>Detail:</b> <ul style="list-style-type: none"> <li>• Decimal practice with base 10 manipulatives</li> <li>• Decimal place value</li> <li>• R/W decimals</li> <li>• Estimating decimals</li> <li>• Add/Subtract Decimals</li> <li>• Multiply/Divide Decimals</li> <li>• Long Division w/ decimals</li> <li>• All decimal operations with story problems</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Dividing fractions by whole numbers</li> <li>• Dividing fractions by fractions</li> <li>• Factor groups</li> <li>• Common factors</li> <li>• GCFs</li> <li>• Distributive property</li> <li>• Multiples</li> <li>• LCMs</li> <li>• Applications</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Ratio intro</li> <li>• Unit rates</li> <li>• Unit rate comparisons</li> <li>• Graphing ratios</li> <li>• Comparing and scaling</li> <li>• Double number lines</li> <li>• Equivalent ratios</li> <li>• Finding percents using percent bars</li> <li>• First quadrant graphing</li> <li>• Tape diagrams</li> <li>• Percent applications</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Combining like terms</li> <li>• Order of operations</li> <li>• Distributive property</li> <li>• Exponents and powers</li> <li>• Number properties</li> <li>• Variables and patterns</li> <li>• Writing expressions</li> </ul>
Unit 5: Equations and Inequalities	Unit 6: Rational Numbers	Unit 7: Area and Volume	Unit 8: Statistical Thinking
<b>Detail:</b> <ul style="list-style-type: none"> <li>• Writing equations</li> <li>• Making an equation true</li> <li>• Solving addition equations</li> <li>• Solving subtraction equations</li> <li>• Modeling multiplication and division equations</li> <li>• Inequalities</li> <li>• Writing inequalities and testing solutions</li> <li>• Inequality applications</li> <li>• Creating tables of two variables</li> <li>• Writing algebraic equations to represent relationship of two variables</li> <li>• Creating and graphing x/y tables and coordinates</li> <li>• Dependent/Independent relationships and variables by comparing table, graph, and equations</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Integers</li> <li>• Integers on number lines</li> <li>• Integer vocabulary in everyday life</li> <li>• Coordinate plane integers</li> <li>• Absolute value</li> <li>• Opposites</li> <li>• Recognizing different signs of similar coordinate pairs will land on different quadrants in the plane</li> <li>• Use vertices on coordinate planes of polygons to find lengths of sides</li> <li>• Plot vertices to draw polygons</li> <li>• Real world applications</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Shapes and designs</li> <li>• Quadrilaterals</li> <li>• Review are of rectangles</li> <li>• Area of parallelogram</li> <li>• Area of triangles</li> <li>• Decompose polygons into rectangles and triangles to find separate areas and combine</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Mean, median, mode, range</li> <li>• Line plot / dot plot</li> <li>• Histogram</li> <li>• Terms: extremes, clusters, gaps, and outliers</li> <li>• Box plots: determine upper and lower extremes, quartiles, measure of variation, range, interquartile range, mean absolute deviation</li> <li>• Compare and contrast line plot, histograms, and box plots</li> </ul>

Table 10

*Seventh Grade Mathematics Grade Level Goals*

Unit 1: Proportional Relationships	Unit 2: Operations with Rational Numbers	Unit 3: Part I: Problem Solving with expressions, equations, and inequalities	Unit 3: Part II: Geometry
<b>Detail:</b> <ul style="list-style-type: none"> <li>• Ratios and Rates</li> <li>• Unit Rates</li> <li>• Unit Rates with fractions</li> <li>• Proportions</li> <li>• Cross Products</li> <li>• Scale drawings</li> <li>• Proportion Application</li> <li>• Proportional relationships &amp; representations of them: table, graph, equation, diagram</li> <li>• Slope</li> <li>• Ratios, Percents, Fractions</li> <li>• Finding Common percents</li> <li>• Percent proportions</li> <li>• Percent increase and decrease</li> <li>• Discounts, markups, tax, tips</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Order of Operations</li> <li>• Comparing, Ordering, Rounding Decimals</li> <li>• Decimal Operations</li> <li>• Simplifying, Comparing, Ordering Fractions</li> <li>• Fraction Operations</li> <li>• Introduction to Integers</li> <li>• Rational Numbers</li> <li>• Adding Rational Numbers</li> <li>• Integer Operations</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Writing expressions and equations</li> <li>• Modeling &amp; Simplifying Expressions with Algebra Tiles</li> <li>• Simplifying Expressions</li> <li>• Equivalent Expressions, Tiling Pools</li> <li>• Area Models</li> <li>• Distributive Property</li> <li>• Writing and Solving One Step Equations</li> <li>• Writing and Solving Multistep Equations</li> <li>• Percent Equations</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Square roots</li> <li>• Area Formulas and Practice</li> <li>• Area of Polygons with missing parts</li> <li>• Circumference of circles</li> <li>• Area of circles</li> <li>• Circle Applications</li> <li>• Area of Irregular figures</li> <li>• 3D Figures and Nets</li> <li>• Surface area of right prisms</li> <li>• Volume of Prisms</li> </ul>
Unit 3: Part III: Inequalities	Unit 4: Geometry	Unit 5: Probability	Unit 6: Statistics
<b>Detail:</b> <ul style="list-style-type: none"> <li>• Inequalities Using Addition, Subtraction, Multiplication, Division</li> <li>• Multi Step Inequalities</li> <li>• Writing, Solving Inequalities with story problems</li> <li>• Inequality Application</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Intro to Angles</li> <li>• Investigation Angles: Complementary, Supplementary</li> <li>• Investigating Vertical and Adjacent Angles</li> <li>• Writing Equations about angles</li> <li>• Investigating Triangles</li> <li>• Triangle Inequality</li> <li>• Triangle Construction</li> <li>• Quadrilateral investigation</li> <li>• Construction of Quadrilaterals</li> <li>• Cross sections of prisms, pyramids, cubes</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Intro to Probability - ratios, rates, collecting data</li> <li>• Making predictions</li> <li>• Collecting data and approximating probability</li> <li>• Equally likely and sample spaces</li> <li>• Lists, tables, tree diagrams</li> <li>• Counting principle</li> <li>• Area models</li> <li>• Relative frequency</li> <li>• Prediction</li> </ul>	<b>Detail:</b> <ul style="list-style-type: none"> <li>• Statistics Introduction and Dot Plots</li> <li>• Histograms</li> <li>• Mean and Variability</li> <li>• Mean Absolute Value Deviation</li> <li>• Median</li> <li>• Box Plots</li> <li>• Interpreting Box Plots</li> <li>• Histograms</li> <li>• Box and Whisker plots</li> <li>• Random sampling</li> <li>• Analyzing data</li> </ul>