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English Learner Advancement in AVMR Structuring

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English Learner Advancement in AVMR Structuring

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By

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Dedication

This thesis is dedicated to my husband, Quentin Bradley Jr., who continuously encourages, supports, and restores my confidence. Your patience and love has simultaneously seen us through both our first three years of marriage and the entirety of grad school. This is our accomplishment together! It is also dedicated to my parents, Rodney and Julie Agnes. They are my greatest models and reminder of the importance of worthy commitments, both personally and professionally. Their love is always abundant.

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Abstract

Add+Vantage Math Recovery (AVMR) is an intensive remedial math program that involves diagnostic assessments and learning trajectories that help teachers identify and work with students who have been identified as having significant difficulties and specific learning gaps in early number learning. When analyzing the AVMR data to see if students can structure numbers as expected for their grade levels in the school district where this study takes place, teachers in the district noted that those who struggle the most are English learners (ELs) and students receiving special education services. While students receiving special education services will continue to experience specialized instruction for their individual learning needs, after one year of AVMR instruction, ELs are expected to advance at the same rate as an average-performing native speaker of English (NSE). The goal of this study was to test that assumption. The study sought to identify a trend at which ELs advance through the levels of structuring numbers in the AVMR intervention program and compare that trend to the trend for native speakers of English in the same grades. The participants of this study were selected based on the amount of continual AVMR structuring instruction they received during grades 2-5. Four years of existing data from AVMR assessments scores for the same group of students who did not receive special education services from a school district in the Midwest region of the United States were analyzed with MANOVA, ANOVA, and descriptive statistics. The results revealed that ELs follow a significantly different trend from their native speaker peers.

Chapter One: Introduction

AVMR (Add+Vantage Math Recovery) is a remedial math program; an intensive program of math instruction for students who have been identified as having significant difficulties in early number learning. The program consists of three courses in which teachers are trained to administer and analyze assessments, then select appropriate activities and approaches to use with students based on the constructivist theory of learning. AVMR Course 1 covers: addition and subtraction, number sequences, numeral identification, and structuring numbers. AVMR Course 2 extends Course 1 and covers: conceptual place value and multiplication and division. Course 3 covers fractions. Currently, the district in which this study took place requires designated teachers to be trained in Course 1 and Course 2, but not in Course 3. The skills and concepts from AVMR Course 1 and Course 2 follow a progression on topics informing teachers of current students' understanding and what should occur next, making planning equally appropriate and challenging for each student in the classroom. (U.S. Math Recovery Council, n.d., "Course 1 & Course 2")

To provide AVMR instruction, teachers participate in intensive training courses (i.e., AVMR Course 1, AVMR Course 2). The AVMR program supports the professional development of teachers for the purpose of improving classroom instruction and student math achievement. It provides these teachers with detailed understanding and a framework of how children develop early numeracy comprehension. It has proven to transform teachers' approaches to math instruction and understanding the conceptualization of early numeracy of children (U.S. Math Recovery Council, n.d., "Math Recovery Research, Data, & Results").

The district in which this project took place began implementing AVMR in June 2011 by having 30 Title 1 math teachers and one to two classroom teachers from each building train on AVMR Course 1 content. In 2012, all classroom teachers of grades K-2 were trained on AVMR Course 1. When that phase of implementation was completed in 2015, three AVMR-focus buildings were identified and all classroom teachers of grades 3-5 in those buildings and some special education teachers were trained on AVMR Course 1. The following year, all classroom teachers of grades 1-5 in those same focus buildings and selected special education teachers were trained on AVMR Course 2 (L. Barlow, personal communication, July 23, 2018).

Presently, one of the sixteen elementary buildings in the district has fully implemented AVMR instruction; four more buildings are in their final phase and will take another one to two-years to achieve full implementation; two buildings are starting year two of implementation; two buildings are starting their first year of implementation; and by August 2018, the district had a Math Recovery Intervention Specialist (MRIS) in training in fourteen of the sixteen elementary schools to support classroom teachers with AVMR implementation. Thus, the AVMR program continues to grow and is of high importance to the district, which is investing great effort and time to use the program with fidelity.

As a mainstream classroom teacher trained on AVMR Courses 1 and 2, the researcher facilitates and monitors the progress of small-sized math groups within her classroom. The small groups include combinations of English learners (ELs) and native speakers of English (NSEs) at the same AVMR levels. When analyzing grade-level AVMR structuring data during math professional learning communities (PLC), student data is organized by AVMR structuring levels to observe student progression and identify outliers who need additional Title I support. Outliers often include ELs and special education students. It has been observed in the researcher's classroom, and in classrooms of colleagues, that ELs who have been in the school for two or more years seem not to advance at expected rates for the amount of AVMR intervention received. For example, said students receive AVMR structuring instruction in both 2nd and 3rd grade and, in some cases, additional Title I support with a teacher providing individual intervention through MRIS, but they do not show expected progress upon assessment. Is this gap a real difference between ELs and NSEs, or is the gap merely an impression created in teachers' minds by a few cases and does not represent an overall pattern? If the gap is real, is it a reflection of teachers' instruction, academic language difficulties, potential math deficiencies, subjective assessment results, past educational experiences or lack thereof, or a combination of the latter? Furthermore, how might the dual requirements of academic language acquisition and content knowledge learning that ELs face affect AVMR structuring progression?

When communicating these thoughts to her building's student performance strategist, the researcher learned that district-level data have shown the first year of AVMR instruction results in limited growth for ELs, but is believed that after one year of AVMR instruction, ELs advance at the same rate as an average-performing NSE. This response indicates that the gap between ELs and NSEs is real, but it appears that there is another gap, this time in the data for the district: current data are informative on ELs after one year of AVMR intervention, but no data exist to inform that district on what happens after one year of intervention. In the absence of data, district administrators must rely on beliefs and speculations rather than research results for decisions.

Research Goal

The goal of this study is to gather data on ELs in the AVMR program and their NSE peers, identify trends at which ELs and their NSE peers advance through the levels of structuring numbers in the AVMR intervention program, and compare the two groups. After analyzing the data, the researcher may provide information to the district on further implementation of the AVMR program for the EL population. Furthermore, this study will add to the existing body of research surrounding AVMR, in particular the effects of the AVMR program on ELs, an area with very minimal research until now.

Definition of terms.

English learners (ELs): persons in school who need to develop English fluency and acquire academic knowledge and language to succeed in educational settings where instruction is entirely in English (Vásquez, Smith, & Hansen, 2013)

Migrant English learners: students from families who follow work from state to state resulting in interruptions in education

Transitional English learners: students who return to home country for portions of the academic year

Bilingual English learners: students who are able to function sophisticatedly or limitedly in more than one language

Children from immigrant families: students who have at least one foreign-born parent

Language minority students: students whose first language is less common or not highly used by others

Structuring numbers: students' facility to combine and partition numbers without counting by ones, but rather by grouping or using five and ten as reference points (Wright, Stanger, Stafford, & Martland, 2012)

Literature Review

English learners are identified because of their need for English language instruction. The AVMR program is designed to identify and instill specific skills and strategies needed at an early age to fully understand and successfully use more complex mathematical strategies. When used properly, the program identifies low-performing and at-risk math students. Features and research of the program do not address language learners' needs specifically or the simultaneous tasks ELs perform during the intervention's instruction—learning math content while acquiring social and academic language. The review of literature explores the major components of the project: ELs, academic language acquisition, assessing ELs, teaching/learning math content, AVMR, and structuring numbers in hopes of drawing conclusions on the rate at which ELs advance through the levels of AVMR structuring.

Defining and identifying English learners. There were over 4,800,000 English learners (ELs) enrolled in schools during the 2014-2015 academic year; that is approximately 10% of the K-12 population (U.S. Department of Education, n.d.). Nearly 97% of the more than 4.8 million ELs received services in a language instruction program (U.S. Department of Education, n.d.). ELs are most commonly identified in schools by administering home language surveys (HLS) (U.S. Department of Education, 2017; Zacarian, 2012). Although not mandated, HLSs may include questions approved by the Office for Civil Rights (OCR) and U.S. Department of Justice (DOJ) that read: "What are the primary languages used in the home regardless of the language spoken by the student?" "What is the language most often spoken by the student?" and "What is the language that the student first acquired?" (U.S. Department of Education, 2017). HLSs are beneficial tools because by law schools are mandated to identify ELs in a timely matter, that is 30 days within the start of the academic year or within two weeks at any other enrollment time

(U.S. Department of Education, 2017). Students who are identified as EL through an HLSs come from a variety of different backgrounds; the population of ELs is diverse in several ways. An EL student may be entering school between the ages of three and 21 with varying formal or informal educational experiences; have been born in or outside of the United States; be a native resident (e.g., Native American, Alaska Native), immigrant, refugee, or migrant; and be the child of parents who do or do not speak English (Zacarian, 2012). ELs represent a variety of socioeconomic, cultural, and linguistic backgrounds. There are multiple terms used to group or identify students with first language(s) that are not English. In this paper, EL will encompass a number of commonly used terms, such as English language learner (ELL), English as a second language (ESL), limited English proficiency (LEP), migrant English learner, transitional English learner, and bilingual English learner, as well as less commonly used or accepted terms, such as children from immigrant families and language minority students (Vásquez et al., 2013; Macías, 2002).

Academic language acquisition. Language that schools help ELs acquire can be placed in two categories: social and academic. Social language is the language used in everyday communications (WETA Public Broadcasting, 2019) and differs from academic language rhetorically, structurally, and in vocabulary (NCTE, 2008). Academic language is more demanding and complex, and necessary for succeeding in a school setting where only the second language is used during instruction (Barrow, 2014). It is imperative that teachers can identify proficiencies in each area of language for ELs. Because academic language is a significant determiner for academic success, it would be beneficial to teachers to be mindful of their students' English language skills prior to completing the AVMR structuring assessment as less proficient students are likely to perform at a lower level due to their gap in English academic language.

Age does not seem to be the main determiner for whether ELs will or will not become academically proficient (Snow & Freedson-Gonzalez, 2003), but rather age is interconnected with ELs' past and present educational settings and experiences (Snow & Freedson-Gonzalez, 2003; Marinova-Todd, Marshall, & Snow, 2000; McLaughlin, 1992, Collier, 1987), socio-economic position (Snow & Freedson-Gonzalez, 2003), immigration status, length of language exposure and practice (Snow & Freedson-Gonzalez, 2003; Marinova-Todd et al., 2000; Collier, 1987), and motivation (Kormos & Csizér, 2014; Snow & Freedson-Gonzalez, 2003; Marinova-Todd et al., 2000; McLaughlin, 1992). All these factors create significant variations in learning processes and in return affect how ELs learn language that is content-specific (NCTE, 2008). Furthermore, acquisition of academic language proficiency results from ELs' home, school, and community influences; metacognitive, metalinguistic, and metacultural awareness; and access to and interaction with complex thinking tasks (WIDA, 2010). Development of the academic language in the first language also influences second language academic learning and success (Barrow, 2014; WIDA, 2010; Marinova-Todd et al., 2000; McLaughlin, 1992; Collier, 1987). ELs who studied for two or three years in their native language took less time to acquire academic language in their second language (Barrow, 2014; Collier, 1987). Certain teaching techniques also produce greater results in ELs' academic content knowledge and language acquisition (Barrow, 2014; McLaughlin, 1992). Because the many variables involved differ for each and every student, the length of time necessary to achieve academic language proficiency will fluctuate anywhere from four to ten years (Sparks, 2016; Barrow, 2014; Collier, 1987). If the trend analysis reveals ELs do not show the expected growth after one year of structuring instruction, but take longer, this study will support the claims of these researchers and add to the body of AVMR research noting that additional time seems necessary for ELs to achieve the academic language necessary for AVMR structuring content. The exact cause for any extended time will remain unknown due to many factors identified here that are entwined with each EL student.

Assessing English Learners' language proficiency. Careful analysis of HLSs assists in identifying students that need further screening and assessment to determine their levels of English language proficiency and whether there is a need for English language instruction. The purpose of assessing and obtaining the proficiency levels of potential English learners may include but isn't limited to placing students in courses, setting goals and expectations for achievement, and informing instructional and program decisions to further improve the education of ELs.

An English language proficiency assessment that 38 U.S. states or territories have adopted is the Assessing Comprehension and Communication in English State-to-State for English Language Learners, or ACCESS for ELs. It is an annual, large-scale assessment that is administered to kindergarten through 12th grade students who have been identified as ELs to assess the four language domains (i.e., listening, speaking, reading writing) and acts as a tool to monitor the progress of ELs in academic English development (WIDA, 2014). It is aligned with the WIDA English Language Development Standards. All ELs in the state in which this study took place are required to complete the ACCESS for ELs assessment. Once the four language domains are individually calculated on a scale of 1.0-6.0, a composite score incorporating the four domains is calculated on a scale of 1.0-6.0. The composite score determines the level at which an EL is placed (i.e., entering 1.0-1.9, beginning 2.0-2.9, developing 3.0-3.9, expanding

4.0-4.9, bridging 5.0-5.9). A composite score of 5.0 or greater, in which individual domain scores are no less than 3.5, determines that an EL where the researcher works is permitted to exit a language program. However, each state sets its own exiting criteria. Descriptions of WIDA performance definitions (WIDA, 2014) can be examined in Appendix A.

It is recommended to use a combination of or multiple assessments for varying purposes (e.g., content knowledge, literacy skills, language acquisition) when making decisions to best determine the social and academic needs and services of ELs (NCTE, 2008; Durán, 2008). There is a variety of other assessments schools can draw on (e.g. classroom assessments, standardized assessments), and most states also allow other factors (e.g., a teacher's clinical judgement, parents' input) to influence the decisions on students' English language instruction (Durán, 2008). The district in which this study took place screens students through the district's HLS. Those identified are evaluated through the WIDA consortium MODEL screener test to see if they meet the criterion to receive EL support services. Daily minutes of EL services varies depending upon each student's needs. Parent notification forms are given to guardians following identification and notifies of the students' English language proficiency (ELP) level and which EL services will be provided. Parents may refuse services by completing a refusal of services form; however, students will continue to be assessed for ELP yearly until a 5.0 exit score is achieved. Annually, all EL students take the standardized WIDA ACCESS test to determine their ELP. An individual language plans (ILP) is written for each EL annually. Students are exited from the EL program when they score a 5.0 on the ACCESS test. Exited students continue to be progress monitored for two years to ensure they are being successful in the mainstream classes. From this established process for assessing and identifying students as ELs, the groups of ELs and NSEs for this study will already exist in the district database of AVMR structuring assessment records.

Effective teaching of mathematics to English Learners. Effective strategies and practices for teaching math to ELs helps to balance the double demands of math content and language learning. Broadly speaking, students should have opportunities and be encouraged and supported for speaking, writing, reading, and listening in math classes because the results lead to greater academic and language success (Doabler, Nelson, & Clarke, 2016; Baker et al., 2014; Bruun, Diaz, & Dykes, 2015; NCTM, 2000). Additionally, effective strategies that support speaking, writing, reading, and listening in math class benefit EL students because they "communicate to learn mathematics, and they learn to communicate mathematically" (NCTM 2000, p. 60). Verplaetse (2014) describes effective strategies in general:

> Language-rich practices are those highly interactive classroom practices which allow students to express academic thoughts in full, extended utterances; require students to express high-cognitive level ideas both orally and in written form; and create interactive, topic-based discussions between teacher and students and among students. (pp. 632-633)

It is also necessary to remember that ELs are diverse students coming from various backgrounds and cultures with varying levels of language proficiency and educational experiences, thus each strategy or method will have differing outcomes. Civil (2014) notes that the culturally laden characteristics of mathematics and mathematics instruction are described at length in the works of Alan J. Bishop, Terezinha Nunes, Analucia Schliemann, and David Carraher, the Diversity in Mathematics Education Center for Learning and Teaching, Norma Presmeg, and Guida de Abreu. ELs' cultures have different ways of *doing* math, whether it may be different algorithms, topics, or ways of teaching. Differences in mathematics language and mathematics education come with ELs whether a teacher is aware or unaware of the differences (Barwell, 2014; Civil, 2014), but when ELs' cultures are considered, teachers can better understand and anticipate possible confusions. By considering culture and using strategies, teachers can better identify true mathematic capabilities of ELs. If culture is not considered, student and teacher misunderstandings may continue and lead to further difficulties in academic language learning and assessing content knowledge.

While effective instruction for ELs provides opportunities for ongoing language development in all four domains and addresses cultural perspectives students bring to the classroom, there are effective techniques for the subject area of mathematics itself that should also be a part of the lessons ELs receive. These include explicit instruction, vocabulary instruction, graphic organizers and note-taking, peer discussions and cooperative learning, visual manipulatives, and think-alouds.

Explicit instruction. Explicit instruction allows for clear, organized opportunities for students to acquire manageable amounts of mathematical vocabulary and unfamiliar words, words with multiple meanings, skills, strategies, and concepts incorporating all four language domains (Baker et al., 2014). A teacher directly modeling these elements with activation of prior knowledge, clear explanations and discussions, and feedback is also key to using explicit instruction effectively when teaching math to ELs. Doabler, Nelson, & Clarke (2016) said that teacher feedback following explicit instruction should be timely, informative, targeted to a concept, corrective as well as positive; include mathematical vocabulary; and draw on prior and current learning experience(s). Teacher feedback may also include modeling and follow-up questions, such as asking "why" and "how" questions to explore further understanding and allowing ELs opportunities to produce mathematical verbalizations (Doabler et al., 2016).

One study by Alt, Arizmendi, and Beal (2014) intended to find implications language has on math performance, and the study concluded that a significant difference in performance was found between ELs and NSEs performances when math tasks were language-heavy and symbol-heavy, but overall not significantly different for other varying language and symbol bearing tasks. Explicit math instruction techniques help alleviate additional cognitive overload. A research study performed by Doabler et al. (2016) found that EL students who received systematic and explicit core mathematic instruction outperformed EL peers in control groups regardless of their prior mathematical knowledge. Many earlier as well as more recent research studies discussed by Hiebert and Grouws (2007) indicate that explicit attention to conceptual essentials are most effective for retention of knowledge and skills.

Vocabulary instruction. Vocabulary needs to be intensively taught through an assortment of activities that span more than a few days. Over the several days, instruction and structured opportunities for practice should involve all four language skills (i.e., speaking, listening, reading, writing) (Baker, et al., 2014). Small group instruction to work with specific problems in development should be designated by the teacher (Sparks, 2016). Unknown contexts and experiences, familiar and unfamiliar word parts, and cognates are all points to address when teaching vocabulary to ELs (Bruun, et. al., 2015) because so much of today's math is high in language content. To organize understanding and allow for opportunities to recall math vocabulary, introducing and practicing a number of strategies is recommended as some terms are easily depicted while others allow explanation. Some strategies to provide further vocabulary practice are embedded in other techniques discussed below.

Graphic organizers and notetaking. Recognizing math phrases, vocabulary words, or a corresponding symbol but not remembering the role it has in math (e.g., divide means to split into equal groups as shown by ÷; *yard* or *foot* have several meanings) is a common difficulty. Having notes, examples, explanations, and drawings to refer to and review offers some resolution to the issue. Examples may include visual vocabulary journals, writing journals for analogies or chunking for schema, personal glossaries, and a modified Frayer model (Sparks, 2016; Bruun et al., 2015; Baker et al., 2014; Barrow, 2014; Alt et al., 2014). Learning is documented by the students as it occurs. As reported by Bruun et al. (2015), students can build further fluency and accuracy in both language and mathematics by sharing their notetaking and writing about math with their peers. Keep in mind that notetaking and writing about math will look different for each EL at each level of language proficiency

Peer discussion and cooperative learning. There is a direct relationship between verbalization in mathematics and increased achievement (Doabler et al., 2016). Usage of academic language between teachers and peers builds fluency and proficiency. Deliberate discussions facilitated by teachers (e.g. connect background knowledge, pose statements, build on responses, ask follow up questions, ask for clarification, rephrase) maximizes associations and, therefore, increases retention of previously learned information and connects existing content knowledge with new content knowledge (Doabler et al., 2016; Koelsch, Chu, & Rodriguez, 2014; Moschkovich, n.d.). Within peer discussions and cooperative learning settings, teachers have the opportunity to observe and provide immediate feedback and focus on math content reasoning attempts, not specifically language accuracy (e.g. pronunciation) (Moschkovich, n.d.). *S*tudents who have the opportunity to practice the four language skills (i.e., speaking, writing, reading, listening) benefit because they are both communicating to learn math and learning to communicate math, and the development of both capabilities helps children be successful with math content (Bruun et al., 2015).

Visuals and manipulatives. The relationship between mathematical representations and abstract symbols is hard. Visual models help formulate connections, brainstorm background knowledge, and create connections with the language domains (Thompson & Rubenstein, 2000). Prior to using visual models, concrete examples with manipulatives assist ELs in demonstrating skills while making connections to the academic language (Doabler et al., 2016). One study found that journaling and peer discussions were not enough for ELs due to limitations of their writing and speaking proficiency levels, so visual depictions and manipulatives that can be seen would allow ELs to still demonstrate their understanding in math (Bruun et al., 2015). Getting up and moving, making gestures, playing games, and working kinesthetically can be useful visuals while teaching and learning and using math content language (Barrow, 2014; Thompson & Rubenstein, 2000).

Think alouds. When teachers think out loud, it makes mathematical thinking and problem-solving processes open and visible to ELs so relationships between concepts and skills can be established (Doabler et. al., 2016). Because verbal instruction is not sufficient for all ELs, when thinking aloud teachers should also incorporate a combination of tools and strategies discussed previously based on students' needs. Having students think out loud (e.g. number talks) is another opportunity to improve fluency and automaticity of mathematical verbalizations.

There is an underlying assumption that any curriculum implementing these effective practices will allow ELs to succeed in mathematics content achievement. However, three research studies suggest that ELs struggle to achieve in math for three key reasons: 1) educational levels of mothers, 2) the language rigor of mathematics, 3) math content knowledge.

There are a number of factors that researchers include when analyzing ELs' academic achievements. Mulligan, Halle, and Kinukawa (2012) report a study that followed a 1998-99 kindergarten class until most of those students were in grade 8 (i.e., 2007). When entering kindergarten, the students were categorized into four groups based on language proficiency scores: 1) primary home language is English, 2) English-proficient at kindergarten entry, 3) English-proficient by end of kindergarten, 4) not English-proficient by end of kindergarten), and other factors included were race/ethnicity, poverty status as of 2007, and mother's highest level of education as of 2007. All students made growth in math by grade 5; however, ELs were continuing to underperform NSEs. Non-Hispanic students who were English-proficient when entering kindergarten scored higher in math than Hispanic peers, who were also English-proficient upon entering. Non-Hispanic ELs who were English-proficient by the end of kindergarten scored higher in math than their Hispanic peers. Students whose home language was English showed no measureable differences by race/ethnicity. Three groups showed that those living in households at or above poverty scored higher in math than their peers living below the poverty level; ELs who were not English-proficient by the end of kindergarten showed no measureable difference in math achievement with their peers in relation to socioeconomic status. The research team found that students whose mothers were more highly educated generally achieved higher scores than students with the least educated mothers. In math, ELs who were or were not proficient by the end of kindergarten and whose mothers had a high school diploma or higher scored higher than EL peers whose mothers did not.

Aside from comparing NSEs and ELs background differences to explain achievement differences, attention to language rigor within math content assessments has also been studied and found of high importance. Alt et al. (2014) determined implications associated with linguistic complexities with sixty-one school-aged NSEs (21), ELs (20), and students with specific language impairment (SLI) (20). Students were given tasks with differing language and mathematical demands: language-heavy, symbol-heavy; language-light (e.g. first language and/or technology used), symbol-heavy; language-light, symbol-light; and language-light, visual working memory heavy. ELs scored lower than NSEs when mathematical tasks were language-heavy, symbol-heavy; although no statistical difference in mathematical performance was found between NSEs and ELs when tasks were language-light. Similar to those language-light results as well as taking into account complex grammatical structures and the inclusion of low-frequency, non-math words when assessing ELs, Martiniello (2009) measured state assessment math tasks and determined more linguistically complex tasks showed high differences in achievement scores. Tasks that would include visuals or representations for ELs to make meaning of language-heavy math tasks when assessing math would decrease that difference.

Mosqueda and Maldonado (2013) analyzed a nationally representative data set provided by the National Center for Education Statistics to determine the relationship between 2,005 Latina/o students' proficiency levels, the math courses the students were taking in high school, and their mathematics assessment outcomes, as scores had not shown improvement over time. They determined that although language proficiency is an important factor, increased math achievement was most effected by students' course-taking. They determined that course placement should not be solely based on students' linguistic proficiency levels, but rather give attention to students' math language register and use both to provide equitable access for course placement and opportunities and exposure in math in order to improve assessment scores. The researchers also suggested teacher training to increase usage of effective teaching strategies previously discussed in the paper (e.g., identify and build on prior knowledge, use visual representations, graphic organizers, models).

These studies provide further evidence that ELs typically underperform NSEs in mathematics, adding to a body of evidence others point to (Alt et al., 2014; Barwell, 2014; NCTE, 2008; Durán, 2008; U.S. Department of Education, n.d.). In addition, they identify two areas that school programs can address in particular to help ELs gain greater success in mathematics: focused development of math content knowledge itself, and addressing the rigor of the English language demands in mathematics lessons. If fact, schools often have programs in place to address the development of math content knowledge. AVMR is one of those programs. A study of it and its impact on ELs will help schools understand whether math content knowledge is a bigger barrier for ELs than the language demands of the math classroom.

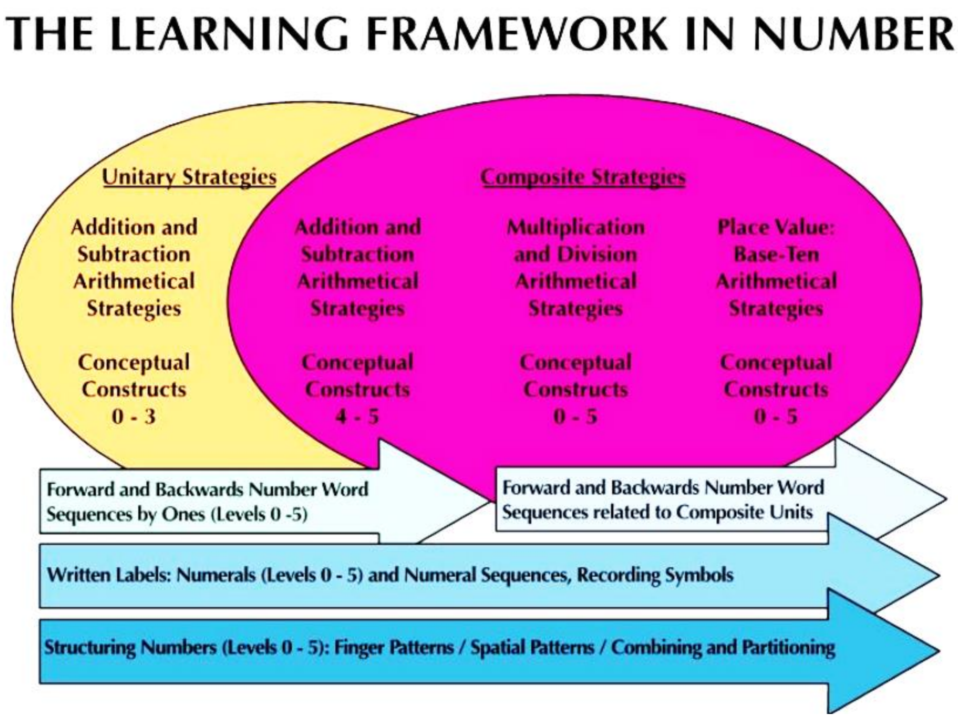
Add+Vantage Math Recovery (AVMR). AVMR is part of a series from Math Recovery developed by Robert Wright, his colleagues, and contributions of notable work from earlier researchers (Tabor, n.d.). The development of Math Recovery is supported by six types of educational research: 1) Foundational Research, 2) Early-Stage or Exploratory Research, 3) Design and Development Research, 4) Efficacy Research, 5) Effectiveness Research, and 6) Scale-up Research (Tabor, n.d.). AVMR follows a progression of topics that has been represented by the LFIN, as shown in Figure 1.

The LFIN depicts constructs (ovals) and the skills supporting the constructs (arrows) necessary to make sense of mathematical concepts (U.S. Math Recovery Council, 2013, p. 16). The framework contains eleven essential aspects of children's early numerical knowledge: 1) Stages of Early Arithmetical Learning (SEAL), 2) base-ten arithmetical strategies, 3) forward number word sequences, 4) backward number word sequences, 5) combining numbers, 6) partitioning numbers, 7) spatial patterns and subitizing, 8) temporal sequences, 9) finger patterns,

10) five-based strategies, and 11) multiplication and division (U.S. Math Recovery Council, 2014, p. 8).

Figure 1

The AVMR Learning Framework in Number. (U.S. Math Recovery Council, 2013, p. 16)



Structuring numbers begins informally at an early age with counting strategies that lead to non-counting strategies. Addition and subtraction were highly researched in the 1980s and early 1990s, and it was found that children typically develop facility with addition and subtraction in interrelated stages that are described in a very similar way to that of Math Recovery's (Ellemor-Collins & Wright, 2009). First, children rely on counting visible objects by ones. Then, children can count visualized objects reciting their counting words. Later, cognitive changes in thinking develop when they can keep track of counting with and then without visualized objects. Children are considered more facile when they do not need to count by ones or track the count using objects or fingers; they realize the meaning behind the number or the objects it represents and use more efficient strategies (e.g., doubles, near doubles, compensation, near 10) or they have automatic knowledge of the combinations and partitions of numbers and parts of wholes (e.g., to make 10 you combine 1 and 9, 2 and 8, 3 and 7, 4 and 6, 5 and 5; to make 7 you combine 1 and 6, 2 and 5, 3 and 4; 13 can be partitioned into 1 and 12, 2 and 11, 3 and 10, 4 and 9, etc.) (Ellemor-Collins & Wright, 2009). Students who are not facile continue to rely on the early strategies (i.e., counting by ones, using fingers or objects) because they see and may be able to write the numbers 8 and 6, for example, but they do not completely comprehend automatically that 8 is more than 6 because of what those symbols actually represent.

The AVMR structuring assessment aims to identify where in the continuum or stages each student has become static, and provides instruction to work towards facility and improve efficiency for structuring numbers to make real-word connections to understanding numbers. Because of its key role in math content knowledge assessment, the results from it will be most informative for determining the success of students in the AVMR program. Any study that hopes to compare ELs to NSEs can use the results of the structuring assessment to draw the conclusions. The AVMR structuring levels are named and described in Appendix B.

AVMR and English Learners. Research has shown that students' understanding about math follows a continuum, similar to literacy (e.g., Reading Recovery) (Tabor, n.d., p. 1; U.S. Math Recovery Council, 2014, p. 2). It is when a student's sense of mathematical concepts and skills is interrupted, or not developed adequately, that correlations between a child's misunderstanding of math and low math achievement or abilities are observed. The students who are low achievers in math tend to remain as that and often develop poor attitudes and give up on mathematics in school (U.S. Math Recovery Council, 2014). U.S. Math Recovery Council, n.d., "Research") reported that students who do not reach upper levels of mathematics (e.g. Algebra) are often enrolled in lower level math classes and are later denied access to four-year colleges. AVMR is designed to allow teachers to locate students' current level of number sense using assessment tools and tasks, and supplement their teaching with intentional instructional strategies to advance students to levels at which they can be successful to increase success as math education continues. There have been many studies conducted using the AVMR program guides. Two studies that include mentions of EL students or students with limited English proficiency will be shared here.

Smith, Cobb, Farran, Cordray, and Munter (2013) conducted a research evaluation experiment from 2007-2009 in 20 elementary schools across five school districts in two states that included 343 students receiving one-on-one intervention using the AVMR program over two years. Of the first graders for year 1, 72.5% of participants received free or reduced lunch, 46.2% were non-white, and 16.3% were limited English proficiency. For year 2, 55.6% of participants receive free or reduced lunch, 46.5% non-white, and 8.8% were limited English proficiency. Effect sizes of the study were found to be between +.30 and +.40 for those who participated in the intervention study, so even though only 10% of the participants received 45-60 of the recommended lessons, there was a positive causal effect of the program. Effects of the program for limited English proficient participants were not reported specifically.

The U.S. Math Recovery Council (2014) presented longitudinal effects of implementing the AVMR program in a Title 1 elementary school. Prior to AVMR implementation, about 30-40% of the students scored proficient or better on the state's standardized mathematics assessment. During the fifth year of implementation, 60% or more students were proficient or advanced in grades 3-5; 76.6% of 5th graders, 82.9% of 4th graders, and 72.8% of 3rd graders. The data also indicate that during the fifth year of implementation, 5th grade students with limited English proficiency were more proficient than other 5th graders in the state, or 76.3% and

36.0% respectively. The county data showed that 65.0% of EL 5th grade students scored proficient or better on the state's standardized math assessment. However, there are no EL data available for the other two grades participating in the study to compare to county and state assessment proficiencies.

When reviewing AVMR literature, the researcher contacted a "Research and Evaluation Specialist" from the U.S. Math Recovery Council and was informed that there are not any data nor an existing study conducted on the "effectiveness of AVMR or MR [Math Recovery] with ELL students specifically" (P. Tabor, personal communication, July 10, 2018)." Thus, a large gap in the literature exists regarding the effectiveness of AVMR for ELs, who may well be identified for inclusion in an AVMR program. The findings of this study will act as the start of a new body of research specifically studying AVMR instruction and assessment and the effect on ELs' mathematical achievements. This study will also add to the body of research encompassing ELs and mathematics assessments. Analysis results may be an indicator that the AVMR structuring assessment is linguistically satisfactory for ELs and that non-linguistic factors (e.g. socioeconomic status, race/ethnicity) are not determiners of success.

Research Questions

The existing body of research reports on the positive effects AVMR instruction has on primary-aged student populations in general, but does not analyze data specifically from ELs. As a teacher who has worked hard to implement AVMR instruction in the classroom and seeks to continue to grow and use the instruction of AVMR to best meet the needs of *every* student, the researcher finds the lack of research noteworthy since the population of ELs enrolled in her district's elementary schools is a significant percentage. In this study, structuring numbers is studied specifically because it is anticipated that fewer ELs receive conceptual place value or multiplication and division instruction due to the LFIN framework that AVMR instruction and the researcher's school follows. Structuring is also the main division of the AVMR program the researcher implements at her grade level. The researcher analyzed AVMR structuring assessment scores with the follow research questions:

- At what rate will ELs advance through the AVMR structuring levels during the first year of instruction compared to NSE peers?
- Will ELs begin to advance through the AVMR structuring levels at a similar rate as their NSE peers after one year of structuring instruction?
- What is the average length of time taken for ELs to reach AVMR structuring level 5 compared to their NSE peers?

By answering these questions, the researcher, the school included in the project, and the school district will have a better understanding of the reach and effects AVMR structuring instruction has had on a specific demographic of the student population. Effective strategies for teaching math to ELs have been studied extensively. Academic acquisition language has been studied extensively. The foundational research used to develop the AVMR program has been studied extensively. Based on the literature surrounding ELs' academic language acquisition and methods for effectively teaching math to ELs, as well as reports indicating significant student growth when the AVMR program is implemented as intended, the researcher anticipates that the data will show a majority of ELs benefiting from AVMR instruction. However, due to educational background differences throughout the EL population, the many variables affecting academic language acquisition that cannot be controlled, and the fact that students who are not proficient in the language of instruction tend to underperform NSE peers, it is also expected that even after one year of AVMR structuring instruction, the majority of ELs will not show advancement through AVMR structuring levels at the same rate as their NSE peers. It is expected that ELs will advance levels, but at a slower rate than their peers.

Limitations

This study will contribute to the lack of research surrounding AVMR specifically pertaining to ELs. However, as the data come from real schools, there are limitations that may affect the results.

The data used is existing; therefore, the collection was not controlled by the researcher and could in turn affect reliability. However, because of the assessment protocols teachers in the district trained in AVMR are instructed to use, it is assumed that the administration of the assessments to collect the data are equal.

Practice effects of the students may vary from year to year as they are assessed by different trained educators from their buildings; students may perform differently at different times of the year depending on their comfortableness within their setting. For example, a student being assessed by their new teacher for the Fall assessment may feel less uncomfortable about sharing their thinking out loud than with that same teacher during the Winter assessment. However, all scores used in this study are Spring scores, which are the final scores reported for the academic year and it is believed to be a respectable indicator of annual growth.

Another limitation the researcher cannot control for are the educational background differences. Students of the EL population may have prior educational experiences or English language exposure to mathematics while others have not, therefore, their informal thinking about mathematics and abilities to verbalize mathematical understandings in English during the assessment may or may not be true indications of math abilities but what is simply observed by the assessor.

If EL students are shown to advance at a similar rate as their NSE peers, this study will add to the limited research surrounding AVMR as an effective program with effective mathematical strategies for teaching structuring numbers to ELs. If the trend does not favor the district's belief that after one year of instruction ELs advance at a similar rate to NSEs, this study may help determine next steps for the district to focus on AVMR teaching strategies for classroom teachers who may not have adequate training for teaching structuring content and language to ELs.

Chapter 2: Methodology

In order to determine a trend for which ELs progress through the AVMR program structuring levels compared to NSEs, AVMR structuring assessment scores of EL and non-EL students who had been instructed and assessed in AVMR structuring over four consecutive academic years were analyzed. Test scores and EL status were subjected to a MANOVA, ANOVA, and descriptive statistics using the IBM SPSS Statistics software program.

Setting

This study was conducted in a school district in a Midwest city of the United States with a population of approximately 122,000 as of 2018. The school district's enrollment of students for the 2018-2019 school year was just over 11,200 in pre-kindergarten through twelfth grade. Approximately 5,400 of those students were in the elementary grades. Those receiving free or reduced lunch in the district were approximately 32%, or 3,600 students. District wide, 73% of students are Caucasian, 15% African American, 5% Asian, 4% Hispanic, 2% American Indian, and 1% Pacific Islander. Languages spoken other than English include Nepali (22%), Somali (13%), Arabic (11%), Spanish (10%), Creoles/Pigin (12%), Bosnian (3%), Swahili (4%), French (2%), Vietnamese (3%), Persian (1%), Dinka (2%), Kinyarwanda (3%), American Indian (1%), Chinese (1%), and Other (12%). Currently, EL students are classified into three categories: Refugee (44%), Immigrant (11%), and Born in the United States (45%).

Three elementary schools from the district this study was conducted in were included and the enrollment numbers for those schools were approximately 490, 330, and 440 students for each academic year from 2015-2018. The average elementary class size is 20 students in each grade level section. There are typically four sections for each grade level. The average family size for each school is 2.77, 2.95, and 2.74. The average household income as of 2016-2017 was $78,000, $44,000, and $60,000, in respect to enrollment numbers and average family size, and the median household income was $60,000, $36,000, and $38,000. These elementary schools were chosen for this study as they are the AVMR-focus schools in the district with either full implementation of the AVMR program or in the last stage for full implementation.

The student populations from these schools make up diverse ethnic groups. Students who resided within the boundaries of the first school were approximately 78.4% White, 10.0% Black, 4.4% Asian, 3.4% Hispanic, 3.4% American Indian, and 0.4% of students identified as Other. Approximately 10.6% of the students who resided in the school's boundaries were identified as having special needs or having an individualized education plan (IEP); 15.8% received services from the English language program; and 48.3% received free or reduced lunches. Of the students attending the second school, approximately 52.1% of the students identified as White, 17.7% Black, 13.0% Asian, 5.6% Hispanic, 5.9% American Indian, and 5.6% of students identified as Other. Approximately 13.5% of attending students were identified as having special needs or having an IEP;19.4% of attending students received services from the English language program; and 70.7% received free or reduced lunches. The third school where participants for this study came from had 69.7% of attending students identify as White, 11.3% Black, 4.9% Asian, 4.6% Hispanic, 3.5% American Indian, and 6.0% of students identified as Other. Approximately 13.3% of attending students were identified as having special needs or having an IEP; 11.1% received services from the English language program; and 45.3% received free or reduced lunches.

Participants

The participants of this study were in grades 2-5 from academic years 2015-2018, between the ages of 7-12, and selected based on the amount of AVMR structuring instruction they received during grades 2-5. The researcher requested the data from the school district's Data Analysis Department to include any student from the three selected schools who had more than one Spring structuring assessment record. The presence of the school district's Spring AVMR structuring assessment record indicates that the student received AVMR structuring instruction for at least part of the year or up to a whole year. The researcher requested student assessment records for the course of four academic years based on the schools' implementation status of AVMR and per district policy that for those students who are ready, AVMR structuring instruction begins in 2nd grade and continues until structuring Level 5 is accomplished. Primary grades end after 5th grade when middle school begins. Middle schools do not implement the AVMR program.

The researcher received 5,929 student AVMR assessment records. Upon conditioning the data, which resulted in the removal of 2,721 records due to lack of clarity in assessment scores or enrollment, 3,208 assessment records were included for a total sample of 928 students. Of 928 students, approximately 57.0% identified as Caucasian, 25.0% Black, 9.0% Asian, 3.7% Hispanic, 3.5% Native American, 0.9% White, 0.8% Pacific Islander, and 0.1% Multi-racial. Demographics were denoted by guardians through district forms. Additional demographic information describing the population includes: 44.1% Female, 55.9% Male, 16.0% ELs, and 84.0% NSEs.

Of the initial 928 students included after conditioning of the data, the final participant count encompassed 255 students having four consecutive Spring structuring scores on record for the four academic years of 2015-2016, 2016-2017, 2017-2018, and 2018-2019. The presence of the four scores indicates that each student was enrolled in the district and received AVMR structuring instruction in grades 2-5. The grade level of the first structuring score (i.e., 2015-2016 academic year) was 2nd grade. The subsequent years and structuring scores are for grades 3, 4, and 5. In 2nd grade the participants' typical age was seven to eight years old. In 5th grade the participants' typical age was ten to eleven years old. Of the 255 students, 20 were ELs. The twelve females and eight males were 50% Asian, 30% Black, 15% Caucasian, and 5% Native American.

Materials

The instrument used in this study is an AVMR assessment. The AVMR assessments are professionally developed, research based, and have been determined to have "high construct validity" (U.S. Math Recovery Council, 2014, p. 10). Determined by the descriptions of the LFIN, the assessments measure students' mathematical understanding and have a "74% agreement when corrected for chance," according to Munn (as cited in U.S. Math Recovery Council, 2014, p. 10). Even with that statistic, it needs to be stated that coding an AVMR assessment can be very subjective and complex. This is why 74% is an impressive number identifying the assessment as reliable. There are protocols assessors must follow during assessment windows to ensure validity of the assessment.

The specific AVMR assessment tool that produced the data used in this study is the AVMR structuring assessment. It is a three-page assessment partitioned into sub-sections that have a number of performance-based tasks to represent the skills for each structuring level, 0-5. It is administered in a one-on-one interview-type setting (U.S. Math Recovery Council, 2013). The assessment is a measurement of participants' math performance and understanding while receiving AVMR structuring instruction. The assessment requires students to respond to regular and irregular dot patterns (e.g., subitizing), demonstrate finger patterns, respond to verbal questioning that includes visuals or tools (i.e., rekenrek), verbal questioning that excludes visuals or tools, and students are frequently prompted to provide further explanation of their mental thought process. The students' responses for this assessment need to be quick, as facilitators are made aware of and practice during the AVMR Course 1 training. The speed indicates proficiency and efficiency of skills.

At the three elementary schools included in this study, all attending students in grades 2-5 are assessed using the AVMR structuring assessment three times a year (Fall, Winter, Spring), or upon new enrollment if outside of one of the assessment windows. This includes EL students who have ILPs and students who receive special education services or have individualized education plans (IEP). Students in grades K-1 may be assessed for structuring based on individual student performance in other AVMR areas.

Assessors record each student's responses on individual assessments. Assessors are also guided to video record assessment sessions to avoid any scoring errors or conflict and later engage in discussions among grade level teams and a MRIS for scoring and instruction input. Scoring the assessment is determined by the objectives described in AVMR structuring Levels 0-5 (see Appendix B). A student is not scored at a particular level until all the skills described for that level have been mastered as demonstrated both in a small group setting and, ultimately, when assessed. For example, in order to be scored as a Level 1, the student must be successful at all tasks in Level 0 and Level 1 as observed both in small group activities and when formally assessed. A scoring tool has been developed and used within the researcher's district that codes the assessment for assistance in scoring students at Levels 0-5. It is replicated from the AVMR structuring assessment and is aligned with the AVMR structuring levels to further prevent scoring errors.

Procedure

The study used existing Spring structuring data records provided by the Data Analysis Department of the cooperating school district. There were 5,929 data records sent to the researcher via district email in a Microsoft Excel file. The data was conditioned based on the number of assessment scores recorded for each student. The researcher removed 2,721 assessment records due to some students having only one record or score, lack of clarity in an assessment score, lack of clarity in an assessment year, or a combination of the latter.

The conditioned data were entered into SPSS for analysis. The records were split into two groups to form the independent variable: those who are EL students (label 1) and those who are NSEs (label 0). Assessment scores were entered for each academic year as Year 1 (2015-2016), Year 2 (2016-2017), Year 3 (2017-2018), and Year 4 (2018-2019). Assessment scores entered matched the AVMR structuring level that scores the assessment. A score of 0 was entered for scoring at structuring Level 0, a 1 for Level 1, a 2 for Level 2, a 3 for Level 3, a 4 for Level 4, and a 5 for Level 5. There were situations where the data were not available to complete each participant's SPSS entry for all four academic years; when the original data file showed that a student was not enrolled for an academic year or did not have an AMVR Spring structuring score, a 10 was entered in SPSS as d*iscrete missing value.*

Data were spot checked during transference and again after transference was complete from the original file into SPSS.

Data Analysis

After data were entered into SPSS, a multivariate test along with descriptive statistics were run to analyze the data. A MANOVA was administered to analyze the dependent variables, or four years of AVMR structuring assessment scores, simultaneously and determine how the independent variables, EL and NSE groups, differ from each other over the course of the four academic years. The $p \leq .05$ level of significance was used for analysis. After the MANOVA was completed, Levene's test was used to determine group variances. The MANOVA produced an ANOVA and additionally, the following descriptive statistics were calculated: mean, standard deviation, and number of cases**.**

Chapter 3: Results

The purpose of this study was to determine how ELs advance through AVMR structuring levels compared to NSE peers; particularly after one year of instruction. It was also to determine how long it takes ELs to reach a structuring Level 5 compared to NSEs. Previous research surrounding ELs' academic language acquisition, teaching math to ELs, and ELs' math achievements suggests results will show that ELs will take more time than after one year of instruction to begin advancing at the rate of a typical NSE. This chapter first presents the results of the MANOVA, then of the ANOVA, and finally the descriptive statistics.

Table 1

Multivariate Tests for Group Effects on Spring Assessments

Effect		Value	F	Hypothesis df	Error df	*p*
Group	Pillai's Trace	.090	6.211[b]	4.000	250.000	.000
	Wilks' Lambda	.910	6.211[b]	4.000	250.000	.000
	Hotelling's Trace	.099	6.211[b]	4.000	250.000	.000
	Roy's Largest Root	.099	6.211[b]	4.000	250.000	.000

a. Design: Intercept + Group

b. Exact statistic

Table 1 provides results on whether or not the groups had an effect on AVMR structuring assessment scores. The four multivariate tests: Pillai's Trace, Wilks' Lambda, Hotelling's Trace, and Roy's Largest Root, are listed in the *Effect* column. Each test's calculated value appears in the *Value* column, *F*-ratios in the *F* column with two degrees of freedom: *Hypothesis df* and *Error df*. Significance values are in listed in the *p* column.

The multivariate tests' calculations provide results on whether or not the groups had an effect on AVMR structuring assessment scores. Each of the multivariate tests use separate linear models that produces each of their values and checks for different variances among the independent variables from the dependent variables. The values produced by each test's different equation is then transformed into an *F*-ratio with two degrees of freedom. The focus is then on the significant values of the *F*-ratios in the *p* column. These results do not indicate how EL and NSE groups differed from one another year to year when assessed using the AVMR structuring assessment in the Spring, but just that statistically significant effects are present. As seen in Table 1, each multivariate test was statistically significant as determined by the *p*-value being less than .05. It is obvious that all statistically significance statistics reported at .01 Alpha. The full table of results for the MANOVA can be reviewed in Appendix C.

Table 2

Results of ANOVA Tests Between-Subject Effects for AVMR Structuring Scores

Source	Spring Assessment	Type III SS	df	MS	F	*p*
Group	Year 1	34.069	1	34.069	16.856	.000
	Year 2	39.552	1	39.552	18.836	.000
	Year 3	6.753	1	6.753	15.143	.000
	Year 4	.019	1	.019	.071	.790

Table 2 provides the ANOVA summary table for the independent variables as one group with effects the group had on each of the dependent variables, or four years of AMVR assessment data, which is produced to determine the nature of the effects discovered from Table 1. Table 2 includes each Spring assessment year, the sum of squares listed in the *Type III SS* column, degrees of freedom in the *df* column, mean square in the *MS* column, *F*-ratio in the *F* column, and the significant values in the *p* column.

There are three sums of squares calculated with their own linear model to produce the values seen in the *Type III SS* column: SS_T, SS_M, and SS_R. The degrees of freedom values are 1, therefore, the mean square values are equal to the calculated sums of squares. The *F*-ratio is calculated and from there it is determined that Years 1, Year 2, and Year 3 for the group were statistically significant, as seen in column *p* of Table 2. *P*-values of less than or equal to .05 were considered statistically significant for this study and all the *p*-values for these years are .01 Alpha. Year 4 was found to not be statistically significant as seen in column *p* where the *p*-value for this year was above .05. The group differences for each year are not yet observable in Table 2. The full table of results for the ANOVA can be reviewed in Appendix D.

Table 3

Results of Descriptive Statistics for AMVR Structuring Scores by Year and EL Status

Spring Assessment	Group	M	SD	N
Year 1	NSE	4.06	1.392	235
	EL	2.70	1.750	20
	Total	3.95	1.465	255
Year 2	NSE	4.31	1.375	235
	EL	2.85	2.159	20
	Total	4.20	1.499	255
Year 3	NSE	4.86	.596	235
	EL	4.25	1.251	20
	Total	4.81	.686	255
Year 4	NSE	4.93	.519	235
	EL	4.90	.447	20
	Total	4.93	.513	255

Table 3 provides separate descriptive statistics on both groups of participants, ELs and NSEs, for the Spring assessment each academic year. For each group, the mean scores are listed in the *M* column, the standard deviations in the *SD* column, and the sample sizes, or number of students, in the *N* column. Table 3 allows the correlation details between the groups' performances and academic years to be observable.

For both groups, only students with four consecutive assessment scores were included. For this reason, column N reveals a large difference in the group sizes; there were 20 ELs and

235 NSEs. When any records were represented as a *discrete missing value,* SPSS excluded the student and their records from the analyses.

As seen in the *M* column of Table 1, the mean scores of NSE students were higher than the mean scores of EL students for Year 1, Year 2, and Year 3. At the end of Year 4, the gap between the mean scores had nearly closed. Appendix B describes the structuring level scores 0-5 from which these means scores were derived. Discussion and implications of the data will be discussed in the next chapters.

Chapter 4: Discussion

This study sought to compare the effectiveness of AVMR for ELs with its effectiveness for their NSE peers. The study examined existing, year-end structuring assessment scores of ELs and NSEs, excluding those having an IEP, over four academic years and analyzed them with a MANOVA, an ANOVA, and descriptive statistics tests. The chapter applies the results of the analyses to the research questions:

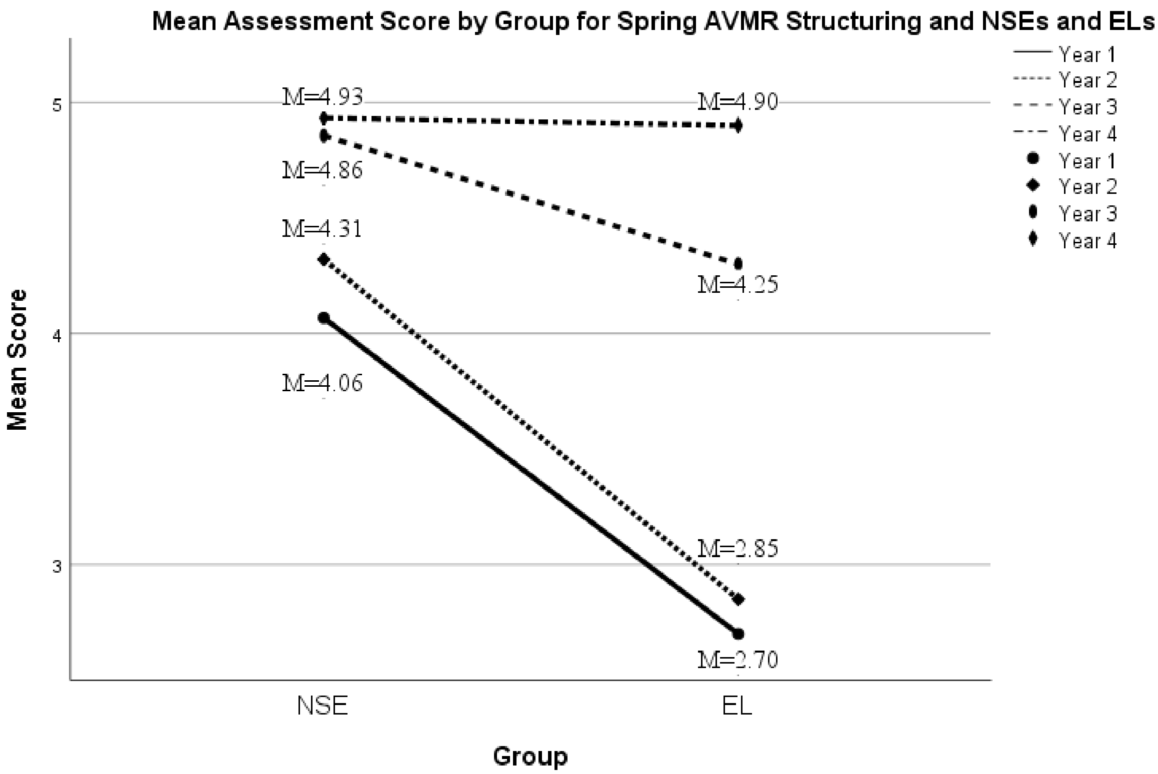
- At what rate will ELs advance through the AVMR structuring levels during the first year of instruction compared to NSE peers?
- Will ELs begin to advance through the AVMR structuring levels at a similar rate as their NSE peers after one year of structuring instruction?
- What is the average length of time taken for ELs to reach AVMR structuring level 5 compared to their NSE peers?

The chapter also identifies additional findings and relates all findings to previous research.

The graph in Figure 2 represents a visual display of the descriptive statistics reported in Table 3 of Chapter 3. The x-axis represents the two independent variables. The y-axis represents the AVMR structuring Levels 0-5. The scale of the y-axis begins just below Level 3 as the lowest mean score between the two groups was 2.7. The difference between each group's mean AVMR structuring score for each Spring is represented by a distinct line pattern with the exact mean score for NSEs labeled on the left and ELs labeled on the right.

Figure 2

The results of the descriptive statistics.



The first research question asked at what rate ELs advance through the AVMR structuring levels during the first year of instruction compared to NSEs. These results can be viewed in Figure 2 and indicate that ELs typically advanced to structuring Level 2 or Level 3 (M=2.70) after the first year of instruction while NSEs typically advanced to Level 3 or Level 4 (M=4.06). The same results were typical after the second year of instruction when M=2.85 (EL) and M=4.31 (NSE). The mean scores indicated that ELs are at lower AVMR structuring levels than NSEs not only during the first year of instruction, as the school district is aware of, but also after the second year of instruction as ELs continued to typically score at Level 2 or Level 3. Being aware of these findings may benefit the schools using AVMR as the teachers may be able to better understand student progress when discussing the growth of individual students and connecting explanations for outliers (i.e., ELs) within their AVMR data. For example, an EL student who began attending an AVMR-focused school in 3rd grade and continued instruction in 4th grade has shown minimal progress after two years of structuring instruction, but aside from many other outside factors teachers can conclude one possible explanation being the student has only had two years of instruction when this study shows a sample size of the EL population not yet progressing after two years of instruction. Being aware of such a pattern can also help teachers and decision makers determine when additional intervention (i.e., Title 1) may be most appropriate and beneficial to student growth.

The second research question asked if ELs will begin to advance through the AVMR structuring levels at a similar rate as their NSE peers after one year of structuring instruction. From the end of Year 1 to the end of Year 2, ELs' mean score changed by +0.15 and NSEs' mean score changes by +0.20. From the end of Year 2 to the end of Year 3 ELs' mean score changed by +1.40 and NSEs' mean score changed by +0.55. From the end of Year 3 to the end of the Year 4 the growth of the ELs' mean score was nine times more than that of NSEs' when ELs' mean score changed by +0.65 and NSEs' mean score changed by +0.07. However, this greater growth of the mean scores after Year 2 and after Year 3 of instruction did not produce higher assessment scores for ELs when compared to NSEs' assessment scores. In fact, the results show that ELs' performance gap with NSEs' increased by the end of Year 2 compared to Year 1; the performance gap between the two groups became greater and differs with the school district's belief that ELs' performance begins to improve after the first year of instruction when the gap in fact increased. The data indicates that ELs did not advance through the AVMR structuring levels at a similar rate as NSEs after one year of structuring instruction. Though there is no previous research to support this, a possible explanation to why this is may be that

that tasks of the assessment are more language and listening heavy as the assessment continues on in assessing the abilities of the students' structuring skills without tools and visuals, which has been found to be more difficult for ELs (Alt et al., 2014).

Results indicated that it was after the third year of instruction rather than after the first or even second year that the mean score for ELs resulted in an immense jump in levels and changed by +1.40, putting both groups of students, on average, scoring at Level 4 when M=4.25 (ELs) and M=4.86 (NSEs). Because the study showed that it was after Year 3 of instruction that ELs began closing the performance gap and after Year 4 that the gap was nearly closed, the findings suggest consistency with the research from Sparks (2016), Barrow (2014), and Collier (1987) that time is needed for academic language acquisition and it typically varies from four to ten years. Research also indicates that many other variables for ELs are present and could be determiners in success, such as cultural adjustment, socioeconomic status, math and language exposure outside of school, etc.

The final research question asked how the average length of time taken for ELs to reach AVMR structuring Level 5 compared to their NSE peers. That answer cannot be provided from this study because after four years of instruction and recording of assessment scores neither groups' mean score reached Level 5 when M=4.90 (EL) and M=4.93 (NSE); however, the mean score of NSEs was nearing Level 5 after Year 3 (M=4.86) while the mean score of ELs was not nearing Level 5 until after Year 4 (M=4.9).

The analysis of test scores of the participants in this study using MANOVA showed a trend over four years suggesting that the AVMR program's structuring instruction has a positive effect on ELs' early understanding of numbers and their relationships, and that more time is necessary for EL achievement in comparison to that needed by English speaking peers as indicated by the statistically significant results found between the groups' mean assessment scores following each of the first three years of AVMR structuring instruction.

Chapter 5: Conclusion

This study found that the AVMR structuring instruction had a positive effect on EL students and that after three years of instruction ELs had begun to perform at the same structuring level as NSEs. The first three years of EL and NSE mean AVMR structuring assessment scores were statistically significant. The fourth year of EL and NSE mean assessment scores were not statistically significant.

These findings are important for stakeholders who currently have implemented or are considering implementing the AVMR program, specifically structuring, into their educational setting where ELs and NSEs will be served alike. The findings of this study provide insight to the impact of AVMR structuring in a way that has not been addressed yet. The findings are, therefore, also important to researchers, who need to analyze the direct effects of AVMR on minority groups.

Limitations

One limitation of this study is the group size of EL students included. The number of assessment records belonging to ELs was substantial; however, not all participants were consistently enrolled in the schools throughout all four years as determined by missing records for particular academic years or student records had missing scores for a number of years, which leads the researcher to believe that the student may not have been assessed. Identifying and including more EL students who were enrolled and assessed consecutively in the same grade levels for the same number of years would allow researchers to analyze a larger sample size.

An additional limitation of the study that is also related to the sample size is the generalization of the study. The students of both groups come from different schools in the same school district that is implementing the AVMR program not all at the same pace. Teachers and specialists working with students in different schools have different lengths of experience and expertise with structuring instruction and that will directly affect structuring work completed with the students at each school. A larger number of structuring assessment scores from across the first four years of instruction from different districts could improve the generalizability, especially assessment scores coming from schools where teachers have been implementing the AVMR program longer resulting in greater teacher expertise.

Another limitation of this study is the inability to identify background differences among the participants that may cause variances between the groups' performances. Differences that may include previous education experiences, amount of math retained in the first language, immigration status, amount of structuring-centered Title 1 support already received, language proficiency, mother's highest educational level, socioeconomic factors, family size, and more. These factors can greatly influence abilities and performance.

A final limitation identified with this study is that only AVMR structuring assessments were analyzed. The AVMR program has several other areas of instruction (i.e., number words and numeral identification, counting forwards and backwards, addition and subtraction, conceptual place value, multiplication and division, factions) that would also need to be analyzed to determine the full effects the program has on the EL population as a whole.

Implications for Practice

Being cognizant of the significant differences after the first three years of instruction and assessment is beneficial to those implementing and using the AVMR program's structuring instruction and assessment practices. Stakeholders who must make decisions about students and math programs should be aware of the anticipated short-term differences in achievement and the long-term effects of the AVMR structuring practices. The findings of this study suggest that it is not after one year of instruction, as believed, that ELs begin advancing at the same rate as NSEs, and this may be helpful as other schools in the district, as well as out of the district, both continue and begin implementation of AVMR. It suggests that decision makers and MRIS teams may consider whether ELs' needs are being met and if more than three years' time of instruction is satisfactory for closing the achievement gap, or if additional development, incorporation and reinforcement of strategies specific for ELs and AVMR is necessary. This study may suggest the timeframe for when additional AVMR structuring intervention with math specialists (e.g., Title 1, MRIS) may be most impactful for lower-performing students, such as ELs, is during the third year of instruction when the group results exhibited a large leap in performance that begins closing the achievement gap.

Implications for Research

The finding that the first three years of AMVR structuring assessments were statistically significant and that ELs began to perform like NSEs on the AVMR structuring assessment at the end of the fourth year has implications for researchers and shows the need for researchers to analyze future studies involving ELs and AVMR to add to this minimal body of research.

Considering there is no additional research available for the implications the AVMR program has on ELs specifically, future research should analyze short term and long term effects to allow further clarifications on its success for ELs. Future short-term and long-term studies may consider analyzing: ELs' performances in AVMR structuring and their language proficiency levels (e.g., WIDA); ELs' performances in AVMR structuring and their background differences (e.g., ethnicity, parent education, previous math education, immigrant status, socioeconomic status); ELs' and NSEs' performance differences in other areas of AVMR (e.g., addition and subtraction, conceptual place value, multiplication and division, fractions); and ELs' performances in AVMR and their performances on state standardized mathematics assessments.

An additional implication for research from this study is the effects the AVMR program has on students who have gaps in their education as this is often the case with ELs for various reasons and the reason for the small sample size in this study.

A final implication for research of this study is the AVMR structuring assessment and the need for researchers to analyze the assessment's tasks complexity for each structuring level in regards to ELs' listening and speaking requirements and how each task correlates to WIDA's language proficiency levels and English Language Development Standards.

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Appendix A

WIDA Performance Definitions

Performance Definitions for the levels of English language proficiency

At the given level of English language proficiency, English language learners will process, understand, produce, or use:

6 Reaching	• specialized or technical language reflective of the content area at grade level • a variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level • oral or written communication in English comparable to proficient English peers
5 Bridging	• the technical language of the content areas; • a variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays, or reports; • oral or written language approaching comparability to that of English proficient peers when presented with grade level material
4 Expanding	• specific and some technical language of the content areas; • a variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related paragraphs; • oral or written language with minimal phonological, syntactic, or semantic errors that do not impede the overall meaning of the communication when presented with oral or written connected discourse with occasional visual and graphic support
3 Developing	• general and some specific language of the content areas; • expanded sentences in oral interaction or written paragraphs; • oral or written language with phonological, syntactic, or semantic errors that may impede the communication but retain much of its meaning when presented with oral or written, narrative or expository descriptions with occasional visual and graphic support
2 Beginning	• general language related to the content areas; • phrases or short sentences; • oral or written language with phonological, syntactic, or semantic errors that often impede the meaning of the communication when presented with one to multiple-step commands, directions, questions, or a series of statements with visual and graphic support
1 Entering	• pictorial or graphic representation of the language of the content areas; • words, phrases, or chunks of language when presented with one-step commands, directions, WH-questions, or statements with visual and graphic support

(WIDA, 2014)

Appendix B

AVMR Structuring Levels and Descriptions

Development of Structuring Numbers

Levels	Brief Description
Level 0 *Emergent*	The student can subitize only small quantities (up to 3) and relies on counting to quantify larger groups. The student builds finger patterns by raising fingers sequentially.
Level 1 *Facile Structures to 5*	The student can subitize regular spatial configurations to 6 and irregular spatial configurations to 5. The student can create finger patterns in the range of 1 to 5 by raising fingers simultaneously. The student is able to combine and partition numbers in the range of 1 to 5 without counting.
Level 2 *Intermediate Structures to 10*	The student utilizes reference numbers involving five-wise (5-plus) and pair-wise (doubles) structures in the range 1 to 10. The student can create simultaneous finger patterns in the range of 6 to 10. The student is able to combine and partition numbers in the range of 1 to 10 in the context of a relevant setting (e.g. 10 frame, arithmetic rack), without counting.
Level 3 *Facile Structures to 10*	The student utilizes reference numbers involving five-wise (5-plus) and pair-wise (doubles) structures to combine and partition numbers in the range of 1 to 10 in the absence of a supportive setting (i.e. bare number tasks), without counting.
Level 4 *Intermediate Structures to 20*	The student utilizes reference numbers in the range 11 to 20 involving a sub-base of 5, the 10-plus aspect of the "teens", and the doubles of 6 through 10. The student is able to combine and partition numbers in the range of 1 to 20 in the context of a relevant setting (e.g. double 10 frame, arithmetic rack), without counting.
Level 5 *Facile Structures to 20*	The student utilizes reference numbers involving 5, 10, doubles, and other structures to combine and partition numbers in the range of 1 to 20 in the absence of a supportive setting (i.e. bare number tasks), without counting.

(US Math Recovery Council, 2013, p. 32)

Appendix C

Complete table of results for Table 1

Multivariate Tests for Group Effects on Spring Assessments

Effect		Value	F	Hypothesis df	Error df	*p*
Intercept	Pillai's Trace	.973	2243.936[b]	4.000	250.000	.000
	Wilks' Lambda	.027	2243.936[b]	4.000	250.000	.000
	Hotelling's Trace	35.903	2243.936[b]	4.000	250.000	.000
	Roy's Largest Root	35.903	2243.936[b]	4.000	250.000	.000
Group	Pillai's Trace	.090	6.211[b]	4.000	250.000	.000
	Wilks' Lambda	.910	6.211[b]	4.000	250.000	.000
	Hotelling's Trace	.099	6.211[b]	4.000	250.000	.000
	Roy's Largest Root	.099	6.211[b]	4.000	250.000	.000

a. Design: Intercept + Group

b. Exact statistic

Appendix D

Complete table of results for Table 2

Results of ANOVA Tests Between-Subject Effects for AVMR Structuring Scores

Source	Spring Assessment	Type III SS	df	MS	F	*p*
Corrected Model	Year 1	34.069[a]	1	34.069	16.856	.000
	Year 2	39.552[b]	1	39.552	18.836	.000
	Year 3	6.753[c]	1	6.753	15.143	.000
	Year 4	.019[d]	1	.019	.071	.790
Intercept	Year 1	842.163	1	842.163	416.663	.000
	Year 2	946.187	1	946.187	450.610	.000
	Year 3	1528.087	1	1528.087	3426.421	.000
	Year 4	1781.697	1	1781.697	6757.084	.000
Group	Year 1	34.069	1	34.069	16.856	.000
	Year 2	39.552	1	39.552	18.836	.000
	Year 3	6.753	1	6.753	15.143	.000
	Year 4	.019	1	.019	.071	.790
Error	Year 1	511.366	253	2.021		
	Year 2	531.248	253	2.100		
	Year 3	112.831	253	.446		
	Year 4	66.711	253	.264		
Total	Year 1	4530.000	255			
	Year 2	5069.000	255			
	Year 3	6014.000	255			
	Year 4	6263.000	255			
Corrected Total	Year 1	545.435	254			
	Year 2	570.800	254			
	Year 3	119.584	254			
	Year 4	66.729	254			

a. R Squared = .062 (Adjusted R Squared = .059)

b. R Squared = .069 (Adjusted R Squared = .066)

c. R Squared = .056 (Adjusted R Squared = .053)

d. R Squared = .000 (Adjusted R Squared = -.004)